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DISCUSSIONS

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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

ECONOMICAL CANAL CROSS SECTIONS

By Victor L. Streeter,[1] Assoc. M. Am. Soc. C. E.

Synopsis

The use of machines for trimming the subgrade and for placing concrete lining in canals has dispensed with the need for simple cross sections composed of straight-line segments. By the use of curved canal cross sections, the hydraulic properties are improved, resulting in a substantial saving in concrete and excavation. This paper shows the method of analyzing several typical curved sections, as well as the most economical trapezoidal section for comparable paving-machine speeds. As the paving machines have cover forms for the concrete which move continuously, their speeds will depend upon the properties of the concrete and upon the steepness of the cross section. An index of steepness is derived in the paper and, assuming the mix to be constant, the speed of the machines is assumed to be constant for cross sections having the same steepness index. This provides a basis for comparing various sections all having the same steepness index.

Introduction

Most of the problems encountered in civil engineering practice require the exercise of judgment and experience. No amount of economic studies will displace this sound judgment and experience, but economic studies of certain factors of an engineering project will provide additional information for the designing engineer which will permit him to design a structure of comparable safety at less total cost.

Large paving machines were used in the construction of the open-channel portions of the Colorado River aqueduct. In 1938, W. L. Chadwick, M. Am. Soc. C. E., and G. E. Archibald, Assoc. M. Am. Soc. C. E.,[2] described this paving machine as follows:

Note.—Written comments are invited for immediate publication: to insure publication the last discussion should be submitted by October 1, 1944.

[1] Associate Prof., Illinois Inst. of Technology, Chicago, Ill.

[2] "Machines and Methods for Canal Construction, Colorado River Aqueduct," by W. L. Chadwick and G. E. Archibald, *Civil Engineering*, February, 1938, p. 104.

"In this machine, the important element was a form or screed plate, shaped exactly to the cross-section of the finished canal, fastened to the under side of a trussed frame, and provided with feed hoppers and a motorized distributor car. * * * The machine, drawn by power winches, operated like a great trowel. * * * The machine was moved forward continuously, shaping both invert and sides as it passed. It was capable of placing lining as accurately as the grade of the tracks [supporting it along the berm] could be maintained. * * * One record shows a combined trimming and placing accuracy of better than 1 per cent."

As the initial cost of a paving machine is large, its use is limited to the construction of long canals of constant cross section. The two principal items of cost are concrete and excavation. The cost of lining is proportional to the wetted perimeter and the cost of excavation depends upon the area, water-surface width, and height of natural ground surface above the hydraulic grade line. The ideal cross section from the standpoint of minimum excavation and concrete would be a semicircle. In general, however, the excavation would not stand, and cover forms for the concrete would be required, making the construction very expensive. By selecting a section which permits the trimmed subgrade to stand by itself, and which permits the use of a paving machine operated at a reasonable speed, a smaller total cost of canal is obtained.

In this paper the speed of paving machine is taken to be a function of the steepness index which is derived in the next section. Several types of cross sections are computed for three values of the steepness index, and their perimeters and water-surface widths are compared.

Notation

The letter symbols used in this paper conform essentially to American Standard Letter Symbols for Hydraulics, prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1942.[3]

The Steepness Index

When concrete is placed on a flat slope (say, 1 on 2) cover forms are not required. The friction developed between the subgrade and the fresh concrete, as well as the adhesion to the reinforcing steel (usually added to control temperature effects), provides stability immediately after placing. There is also some resistance to change of shape in the concrete itself before the initial set. When concrete is placed on a steep slope the effect of friction and internal resistance may not be sufficient to hold the concrete in place. An index to this stability is derived as follows:

Consider a strip of concrete of unit width placed on a sloping surface (see Fig. 1). The force tending to cause motion is the component of the weight tangent to the surface. Let the frictional force resisting motion (including effect of reinforcing bars) be designated by F per unit area of surface; and let γ_c equal the unit weight of concrete. Then for a volume element $t\,dl$ the resultant force tending to cause motion is $t\,dl\,\gamma_c \sin\theta - F\,dl$, provided the first term is greater than the second term. The accumulated force for the

[3] ASA—Z10.2—1942.

slope would be $\int (t\,\gamma_c \sin\theta - F)\,dl$, integrated over the portion of one side of the cross section where the integrand is positive. Taking θ_0 as the angle for which the integrand is zero,

$$F = t\,\gamma_c \sin\theta_0 \qquad (1)$$

and the accumulated force may be expressed as $t\,\gamma_c \int (\sin\theta - \sin\theta_0)\,dl$, carried out from $\theta = \theta_0$ to the water surface. As $\sin\theta = \frac{dy}{dl}$ the force becomes $t\,\gamma_c\,(y_0 - l\sin\theta_0)$, in which y_0 is the vertical distance from the point on the section where $\theta = \theta_0$ to the water surface, and l is the length of section from $\theta = \theta_0$ to the water surface, as shown in Fig. 2. These calculations are for

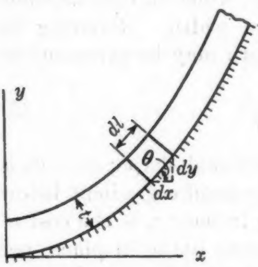

FIG. 1.—SECTION OF CANAL LINING

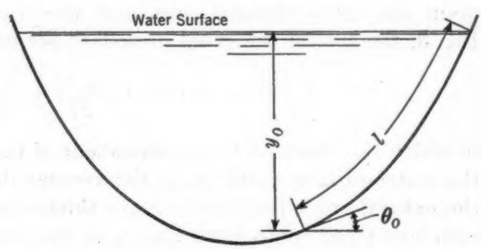

FIG. 2.—A CURVED CANAL CROSS SECTION

sections which are concave upward, or straight lines, which are the only practical shapes for canal cross sections. Dividing the accumulated force by $t\,\gamma_c$, a steepness index, σ, is obtained having the dimensions of a length, thus

$$\sigma = y_0 - l\sin\theta_0 \qquad (2a)$$

If it is considered, for example, that $\tan\theta_0 = 2/3$, then the steepness index becomes

$$\sigma = y_0 - 0.555\,l \qquad (2b)$$

In this paper two canal sections having the same values of σ are presumed to be capable of construction with the same speed of paving machines.

RELATIVE COST OF LINING AND EXCAVATION

The horizontal and vertical alinement of the canal is assumed to have been determined by other studies.[4] The problem of determining the most economical cross section to convey a given discharge at a given slope of the hydraulic gradient for a given or assumed roughness of lining is approached in the following manner.

The Manning formula may be written

$$A\,R^{2/3} = \frac{Q\,n}{1.486\,S^{0.5}} = K \qquad (3)$$

in which: A is the cross-sectional area of canal below water line in square feet; R is the hydraulic radius of the cross section—that is the cross-sectional area

[4] "Aqueduct Size and Slope," by Julian Hinds, *Engineering News-Record*, January 28, 1937, pp. 113–120; and "Economic Sizes of Pressure Conduits," by Julian Hinds, *ibid.*, March 25, 1937, pp 443–449.

divided by the wetted perimeter; Q is the discharge in cubic feet per second; n is the Manning roughness factor; S is the slope of water surface for uniform, steady flow; and K is a known constant for any given study, defined by Eq. 3. Writing $R = \frac{A}{P}$, in which P is the wetted perimeter in feet, Eq. 3 may be written

$$A = P^{0.4} K^{0.6} \qquad (4)$$

The cost of concrete lining for comparable cross sections (same steepness index σ and same freeboard) is proportional to the wetted perimeter, P. For minimum cost of lining, a solution of Eq. 4 is required for minimum P. The cost of excavation depends in general upon the yardage, which in turn depends upon the cross-sectional area and the water-surface width. Referring to Fig. 3, the cost of the canal in dollars per foot of length may be expressed as

$$c_l = C + b_w \frac{y_z c_e}{27} + \frac{A c_e}{27} + \frac{P t c_c}{27} \qquad (5)$$

in which the constant C is independent of the cross-sectional shape; and: b_w is the water-surface width; y_z is the average depth of hydraulic gradient below the natural ground surface; t is the thickness of lining in feet; c_e is the cost of excavation per cubic yard; and c_c is the cost of concrete lining in place, per cubic yard.

In Eq. 5, A, P, and b_w are factors dependent upon the cross section. The problem resolves itself into selecting a cross section for a predetermined steepness index which makes Eq. 5 a minimum. Theoretically, the problem could

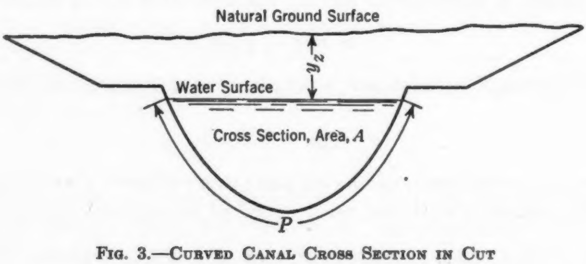

FIG. 3.—CURVED CANAL CROSS SECTION IN CUT

be solved by methods of calculus of variations, but practically it reduces to an examination of the properties of various types of cross sections.

COMPUTATION OF PROPERTIES OF CANAL CROSS SECTIONS

Typical procedures to use in the computation of canal cross-section relationships, such as perimeter, area, water-surface width, and steepness index, are illustrated for the following sections: (1) Trapezoidal cross section, (2) straight sides, circular bottom cross section, (3) circular segment cross section, (4) parabolic cross section, (5) semicubical parabola cross section, and (6) catenary cross section.

The problem reduces to one of finding the dimensions of the cross section such that the given discharge may be conveyed at the specified slope and with a given steepness index. For the first two types of cross sections these conditions

may be satisfied in many ways; hence the condition of minimum perimeter of section is also used. For the first two sections the conditions to be satisfied are

$$(a)\ A\ R^{2/3} = \frac{Q\ n}{1.486\ S^{0.5}} = \text{a known constant} = K \qquad (6)$$

$$(b)\ \sigma = y_0 - l \sin \theta_0 \qquad (2a)$$

and

(c) P = a minimum, subject to conditions (a) and (b) (see Eqs. 2a and 6).

For the remaining four sections, only conditions (a) and (b) (Eqs. 2a and 6) are to be satisfied.

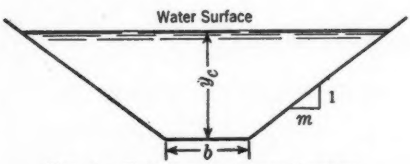

FIG. 4.—TRAPEZOIDAL CROSS SECTION

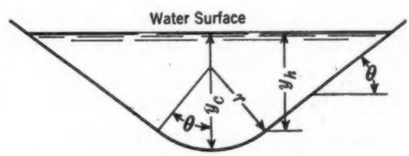

FIG. 5.—STRAIGHT SIDES AND CURVED BOTTOM, CIRCULAR

(*1*) *Trapezoidal Cross Section.*—Using the notation of Fig. 4,

$$A = b\ y_c + m\ y^2{}_c \qquad (7a)$$

$$P = b + 2\ y_c\ \sqrt{1 + m^2} \qquad (7b)$$

$$\sigma = y_c\ (1 - \sqrt{1 + m^2}\ \sin \theta_0) \qquad (7c)$$

and

$$A - K^{0.6}\ P^{0.4} = 0 \qquad (8)$$

Eliminating A, b, and y_c from Eqs. 7 and 8,

$$\frac{P\ \sigma}{1 - \sqrt{1 + m^2} \sin \theta_0} + \frac{(m - 2\ \sqrt{1 + m^2})\ \sigma^2}{(1 - \sqrt{1 + m^2} \sin \theta_0)^2} - K^{0.6}\ P^{0.4} = 0 \qquad (9)$$

which expresses the perimeter, P, as a function of m for given values of σ, θ_0, and K. For minimum P, Eq. 9 is differentiated with respect to m, and $\frac{dP}{dm}$ is set equal to zero. The resulting equation is

$$\frac{P}{\sigma} = \frac{2}{\sin \theta_0} - \frac{\sqrt{1 + m^2}}{m \sin \theta} + \frac{4\ \sqrt{1 + m^2} - 2\ m}{1 - \sqrt{1 + m^2} \sin \theta_0} \qquad (10)$$

which is independent of K. Eliminating P in Eqs. 9 and 10 leaves an equation with m the only unknown. Its value may be determined by trial for given values of σ, θ_0, and K.

(*2*) *Straight Sides, Circular Bottom Cross Section.*—This cross section, shown in Fig. 5, is completely specified by values of θ, r, and y_h. In terms of these quantities (notation shown in Fig. 5)

$$A = y^2{}_h \cot \theta + y_h\ r \times 2 \sin \theta + r^2\ (\theta - \sin \theta \cos \theta) \qquad (11a)$$

$$P = y_h \times 2 \csc \theta + r \times 2\ \theta \qquad (11b)$$

and

$$\sigma = y_h \left(1 - \frac{\sin \theta_0}{\sin \theta}\right) + r\,(\cos \theta_0 + \theta_0 \sin \theta_0 - \cos \theta - \theta \sin \theta_0) \ldots (11c)$$

Using the Manning formula, condition (*a*), it may be shown that the condition for minimum P is

$$y_h = r \cos \theta \ldots\ldots (12)$$

which states that the center of curvature of the circular portion lies in the water surface, and that the result is independent of θ_0 or K. To obtain Eq. 12, eliminate A between Eqs. 6 and 11*a*. Considering y_h and r as functions of P, σ, and θ, differentiate with respect to σ, holding θ constant, and set $\frac{dP}{d\sigma} = 0$. Differentiating Eq. 11*b* with respect to σ and setting $\frac{dP}{d\sigma} = 0$ the terms $\frac{dy}{d\sigma}$ and $\frac{dr}{d\sigma}$ may be eliminated. Simplifying, it is found that $y_h = r \cos \theta$. Eq. 11*c* is not used. Using Eq. 12:

$$A = r^2\,(\theta + \cot \theta) \ldots\ldots (13a)$$

$$P = 2\,r\,(\theta + \cot \theta) = \frac{2\,A}{r} \ldots\ldots (13b)$$

$$\sigma = r \left[\cos \theta \left(1 - \frac{\sin \theta_0}{\sin \theta_0}\right) + \cos \theta_0 + \theta_0 \sin \theta_0 - \cos \theta - \theta \sin \theta_0\right] \ldots (13c)$$

and

$$r = \frac{\sigma + \frac{P}{2} \sin \theta_0}{\theta_0 \sin \theta_0 + \cos \theta_0} \ldots\ldots (14)$$

Eliminating A and r in Eqs. 6, 13*b*, and 14,

$$P^{0.6}\left(\sigma + \frac{P}{2} \sin \theta_0\right) = 2\,(\theta_0 \sin \theta_0 + \cos \theta_0)\,K^{0.6} \ldots\ldots (15)$$

from which P may easily be obtained by trial for given values of σ, θ_0, and K. With P known, Eqs. 14, 6, 13*a*, and 12 yield values of r, A, θ, and y_h, respectively. The water-surface width is given by

$$b_w = 2\,r \csc \theta \ldots\ldots (16)$$

(*3*) *Circular Segment Cross Section.*—The properties of this cross section are completely specified by r and θ, as shown in Fig. 6. Thus

$$A = r^2\,(\theta - \sin \theta \cos \theta) \ldots\ldots (17a)$$

$$P = 2\,r\,\theta \ldots\ldots (17b)$$

in which θ is expressed in radians, and

$$\sigma = r\,(\cos \theta_0 + \theta_0 \sin \theta_0 - \theta \sin \theta_0 - \cos \theta) \ldots\ldots (17c)$$

Using Eq. 6 (condition (*a*)), sufficient equations are available to determine r

and θ for given values of σ, θ_0, and K. This is done by trial from the relations

$$P = \frac{2\,\theta\,\sigma}{\cos\theta_0 + \theta_0 \sin\theta_0 - \theta\sin\theta_0 - \cos\theta} \quad\text{.........}(18a)$$

and

$$\frac{P^2}{4\,\theta^2}(\theta - \sin\theta\cos\theta) - K^{0.6}\,P^{0.4} = 0 \quad\text{.............}(18b)$$

which are derived from Eqs. 6 and 17.

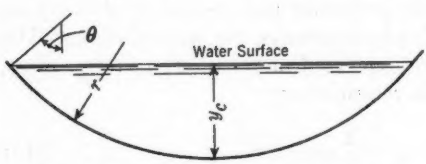

FIG. 6.—CIRCULAR SEGMENT CROSS SECTION

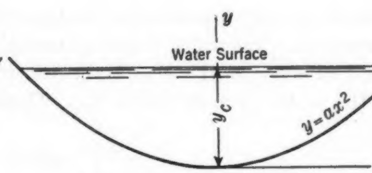

FIG. 7.—PARABOLIC CROSS SECTION

(*4*) *Parabolic Cross Section.*—Referring to Fig. 7, the properties of the cross section are determined for given values of the constant a in the formula

$$y = a\,x^2 \quad\text{........................}(19)$$

and the depth y_c at the center. In terms of y_c and a,

$$A = \frac{4\,y^{1.5}{}_c}{3\,a^{0.5}} \quad\text{.......................}(20a)$$

$$P = \sqrt{\frac{y_c}{a} + 4\,y^2{}_c} + \frac{1}{2\,a}\left[\log_e 2\,\sqrt{a\,y_c} + \sqrt{1 + 4\,a\,y_c}\right] \quad\text{.....}(20b)$$

and

$$\sigma = y_c + \sin\theta_0\left[\frac{1}{4\,a}\log_e(\tan\theta_0 + \sec\theta_0) - \frac{P}{2}\right] \quad\text{.......}(20c)$$

In conjunction with Eq. 6 (condition (*a*)) for specific values of a, K, and θ_0, values of y_c are found from Eqs. 20*a*, 20*b*, and 6. The corresponding values of σ may then be determined by Eq. 20*c*. Different values of a are assumed until the required value of σ is obtained.

(*5*) *Semicubical Parabola Cross Section.*—The computation procedure for the semicubical parabola

$$y = \alpha\,x^{3/2} \quad\text{........................}(21)$$

is similar to that for the parabola. Referring to Fig. 8, the area, perimeter, and steepness index are:

$$A = 1.2\,\frac{y^{5/3}{}_c}{\alpha^{2/3}} \quad\text{......................}(22a)$$

$$P = \frac{0.5926}{\alpha^2}\left[\sqrt{(1 + 2.25\,\alpha^{4/3}\,y^{2/3}{}_c)^3} - 1\right] \quad\text{...........}(22b)$$

and

$$\sigma = y_c - \frac{8}{27}\,\frac{\tan^3\theta_0}{\alpha^2} + \sin\theta_0\left[\frac{0.2963}{\alpha^2}(\sec^3\theta_0 - 1) - \frac{P}{2}\right] \quad\text{....}(22c)$$

With known values of K and θ_0, an α is assumed, and σ is computed, first by

determining y_c and P from Eqs. 6, 22*a*, and 22*b* by trial, and then substituting in Eq. 22*c*. This procedure is continued for several values of α, and the curve α versus σ is plotted. From this curve the value of α for an arbitrary σ may be obtained.

(6) *Catenary Cross Section.*—The particular catenary curve

$$y = \frac{1}{\beta}(\cosh \beta x - 1) \qquad (23)$$

as shown in Fig. 9 is selected so that the perimeter may be obtained easily by integration. Values of β and y_c completely determine the cross section. The area, perimeter, and steepness index are expressed more simply in terms of x_0, half the water-surface width b_w. These relations are

$$A = \frac{2\,x_0}{\beta}\cosh \beta\, x_0 - \frac{2}{\beta^2}\sinh \beta\, x_0 \qquad (24a)$$

$$P = \frac{2}{\beta}\sinh \beta\, x_0 \qquad (24b)$$

and

$$\sigma = \frac{1}{\beta}[\cosh \beta\, x_0 - \sin \theta_0 \sinh \beta\, x_0 - \cos \theta_0] \qquad (24c)$$

From Eqs. 24 and 6, σ may be determined by trial for assumed values of β when K and θ_0 are known. Plotting σ against β, the values of β are determined for a predetermined σ.

Evaluation of Properties of Cross Sections for a Special Case

In this section, Eqs. 6 to 24 are applied to a specific problem, in which $K = 4{,}150$; and $\theta_0 = \tan^{-1} 2/3$. If a slope of 1 on 1.5 is the maximum upon

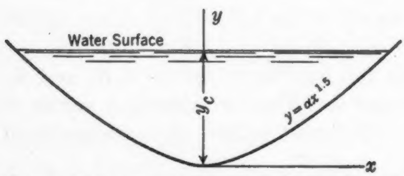

Fig. 8.—Semicubicle Parabola Cross Section

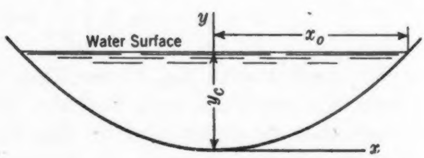

Fig. 9.—Catenary Cross Section

which any length of the concrete may be placed without danger of slumping, the steepness index is given by (see Eq. 2*a*):

$$\sigma = y_0 - \frac{l}{\sqrt{3.25}} \qquad (25)$$

in which y_0 is the vertical distance from the water surface to the point where the cross-section slope is 1 on 1.5, and l is the length along the perimeter of the cross section from this point to the water surface. The six cross sections are determined for $Q = 3{,}000$ cu ft per sec, with the canal slope $S = 0.00004$ and a value of canal roughness $n = 0.013$. These values result in $K = \frac{Q\,n}{1.486\,S^{0.5}} = 4{,}150$ (see Eq. 3). Three values of steepness index are

used for each section: σ = 0.75, 1.50, and 2.25. The equations for the curvature of sections 4, 5, and 6 are given in Table 1. The various properties of the cross sections are given in Table 2.

TABLE 1.—MODIFIED EQUATIONS FOR CURVED CROSS SECTIONS IN ILLUSTRATIVE EXAMPLE

Q = 3,000 cu ft per sec; S = 0.00004; n = 0.013; K = 4,150; and $\theta_0 = \tan^{-1} 2/3$

See Table:	σ	Section 4, Eq. 19 and Fig. 7	Section 5, Eq. 21 and Fig. 8	Section 6, Eq. 23 and Fig. 9
2*a*	0.75	$y = 0.01166\, x^2$	$y = 0.09234\, x^{3/2}$	$y = \frac{1}{0.0211} (\cosh 0.0211\, x - 1)$
2*b*	1.50	$y = 0.01456\, x^2$	$y = 0.10584\, x^{3/2}$	$y = \frac{1}{0.026} (\cosh 0.026\, x - 1)$
2*c*	2.25	$y = 0.0175\, x^2$	$y = 0.1187\, x^{3/2}$	$y = \frac{1}{0.03083} (\cosh 0.03083\, x - 1)$

TABLE 2.—PROPERTIES OF CANAL CROSS SECTIONS; ILLUSTRATIVE EXAMPLE

Section No.	Type; see Fig.:	Area A, in sq ft	Depth y_c at center, in ft	Radius r of circular bottom, in ft	Water-surface width b_w, in ft	Wetted perimeter P, in ft	Slope of section at water surface
			(*a*) σ = 0.75				
1	4	875.8	20.40	(13.97)[a]	71.89	84.82[b]	1.42
2	5	872.4	20.77	20.77	71.58	84.00[b]	1.40
3	6	890.2	16.15	57.46	79.95	88.42	1.03
4	7	880.7	17.20	....	76.81	86.10	1.12
5	8	870.5	20.07	....	72.30	83.67	1.20
6	9	883.5	16.86	....	77.75	86.77	1.09
			(*b*) σ = 1.50				
1	4	870.14	20.34	(15.57)[a]	69.97	83.50[b]	1.34
2	5	865.35	21.01	21.01	69.12	82.36[b]	1.31
3	6	870.08	17.12	47.80	73.26	83.47	0.84
4	7	863.39	18.28	....	70.86	81.94	0.97
5	8	860.02	21.04	....	68.12	81.13	1.08
6	9	865.14	17.88	....	71.57	82.35	0.93
			(*c*) σ = 2.25				
1	4	864.7	20.30	(17.14)[a]	68.02	82.24[b]	1.25
2	5	858.2	21.27	21.27	66.56	80.69[b]	1.20
3	6	854.8	17.88	41.23	67.99	79.93	0.69
4	7	851.6	19.25	....	66.34	79.13	0.86
5	8	852.5	21.91	....	64.84	79.39	0.98
6	9	852.3	18.80	....	66.85	79.33	0.82

[a] Bottom width of trapezoid. [b] Minimum wetted perimeter for the types indicated.

As a means of illustrating the method of comparing sections, assume the cost of concrete lining in place to be c_c = \$15.00 per cu yd, the cost of excavation to be c_e = \$0.15 per cu yd, and the lining thickness, t, to be 0.33 ft. Substituting into Eq. 5 the values of A, b_w, and P obtained for the cross sections for σ = 0.75 (Table 2(a)), it is seen that section 5, the semicubical parabola,

costs less than the others for y_z less than 18 ft. Section 2, with straight sides and circular bottom, costs less than the others for y_z greater than 18 ft. Since y_z enters linearly into the cost equation its average over the length of canal in cut may be used.

For $\sigma = 1.50$ (see Table 2(*b*)), substitution of A, b_w, and P into Eq. 5 shows that section 5, the semicubical parabola, is most economical for all values of y_z. For $\sigma = 2.25$ (Table 2(*c*)), section 4, the parabola is most economical for y_z less than 6.4 ft, whereas section 5, the semicubical parabola, is most economical for y_z greater than 6.4 ft.

Although these comparisons take into account the more important factors in selection of a cross section, there are other factors which should be carefully considered by the designing engineer. For strength against uplift the radius of curvature at the vertex would be important. The average velocity for partial capacities would affect the deposition of sand and silt. In some cases the cost of excavation might be more costly for the deeper cross sections. All of these factors cannot be evaluated in terms of dollars, but should be given careful consideration before the final cross section is decided upon.

Conclusions

With the availability of paving machines for the lining of long canals, curved canal cross sections may be placed at no greater expense than the customary trapezoidal sections. In addition, if a canal cross section is selected which has less water-surface width and less perimeter than the trapezoidal section, both excavation and concrete yardages will be reduced. The most economical canal cross section from the standpoint of concrete yardage is a semicircle. This section, however, could not be used generally because the excavation would not retain this shape, and the concrete would require a fixed cover form.

In order to set up a criterion that would allow the comparison of various sections, a "steepness index" was developed. Two cross sections having the same value of steepness index are assumed to require the same speed of paving machine.

Six particular types of cross sections were investigated to demonstrate methods of determining the properties of sections with a given value of steepness index. The general methods employed will apply to different cross sections and to other capacities than those used in the example. If the same six types of sections were analyzed for a smaller channel, it is likely that different types would prove to be most economical.

Acknowledgments

The study of economical canal cross sections was first undertaken by the writer while employed at the headquarters office of the International Boundary Commission, United States and Mexico, United States Section, at El Paso, Tex. The work was under the supervision of Richard Stephens, Assoc. M. Am. Soc. C. E., and under the general direction of J. L. Burkholder, M. Am. Soc. C. E. L. M. Lawson, Hon. M. Am. Soc. C. E., is the United States Commissioner. The writer wishes to express his appreciation for their aid and suggestions in making this study.

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PAPERS

PRESSURE ON THE LINING OF CIRCULAR TUNNELS IN PLASTIC SOILS

By D. P. Krynine,[1] M. Am. Soc. C. E.

Synopsis

No attempt to develop a mathematical theory of pressure on the circular tunnel lining in plastic soils is made in this paper. Available field data are interpreted, using simple statics, arithmetic, and some geometry. The discussion is concerned mostly with the analysis of pressures on the lining of the Lincoln Tunnel, in New York, N. Y.

Introduction

Important observations of pressure on the lining of circular tunnels have been made in the United States since 1930. These observations represent a genuine contribution of American engineers to the world tunnel literature since, to the writer's knowledge, no such observations on full-sized circular tunnels have been made before. In this paper the available data concerning pressure on the lining of a circular tunnel are analyzed mostly as applied to the Lincoln Tunnel, New York City.[2] A comparison is made between pressures on this tunnel and the Central Avenue Pipe Tunnel,[3] in the City of Detroit, Mich. Finally, a few data referring to the Chicago (Ill.) Subway[4] are also cited. For the sake of brevity, these tunnels will be termed hereafter "Lincoln Tunnel," "Detroit Tunnel," and "Chicago Subway," respectively. The general data concerning these tunnels are as follows:

Diameter.—The outside diameter of the lining of the Lincoln Tunnel is 31 ft, that of the Detroit Tunnel about 13 ft, and that of the tubes of the Chicago Subway about 25 ft.

Lining.—The lining of both the Lincoln Tunnel and the Chicago Subway tubes consists of the conventional cast-iron segments bolted together. The

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **October 1, 1944.**

[1] Research Associate in Soil Mechanics, Dept. of Civ. Eng., Yale Univ., New Haven, Conn.

[2] "Lincoln Tunnel. The Field Measurements and Study of Stresses in Tunnel Lining," by G. M. Rapp and A. H. Baker, The Port of New York Authority, July, 1937 (lithoprinted).

[3] "Earth Pressure on Tunnels," by W. S. Housel, published as part of the Symposium on "Earth Pressure and Shearing Resistance of Plastic Clay," *Transactions*, Am. Soc. C. E., Vol. 108 (1943), p. 1037.

[4] "Shield Tunnels of the Chicago Subway," by Karl Terzaghi, *Journal*, Boston Soc. of Civ. Engrs., Vol. 29 (1942), No. 3, p. 163.

Detroit Tunnel tube is of reinforced concrete. This is a thick pipe, its inside diameter being 9.5 ft.

Soil Conditions.—The average depth of the Lincoln Tunnel below the bed of the Hudson River is about 46 ft. Silt material (organic clayey silt), in which this tunnel is built, is quite similar to that found in the deltas of many other rivers flowing into the Atlantic. This is a plastic material with high natural moisture content. The silt layer in some parts of the Hudson River gorge is perhaps 400 ft thick.

The soil, in which both the Detroit Tunnel and the Chicago Subway are built, is plastic clay. The Detroit Tunnel is located about 60 ft below the ground surface. There are about 12 ft of artificial fill and natural sand covering the soft and medium blue clay deposit, which is about 86 ft thick.

The Chicago soil profile is formed by a layer of silt, sand, and artificial fill, from 10 to 15 ft thick, underlain by a soft blue clay deposit (average thickness 70 ft). Hardpan and rock follow.

Methods of Construction.—Both the Lincoln Tunnel and that part of the Chicago Subway which is referred to in this paper have been constructed by the shield method. The presence of the second tube (in both cases) practically did not influence the results of measurements of the pressure on the lining. The Detroit Tunnel had an open-front face during the construction.

Behavior of the Soil during Construction of the Lincoln Tunnel

Material Above the Haunches and in Front of the Bulkhead.—There is a tendency for the earth in front of the advancing bulkhead to push forward horizontally; but it cannot move in that direction, because of the passive resistance of the earth mass. It must move somewhere, however, since the

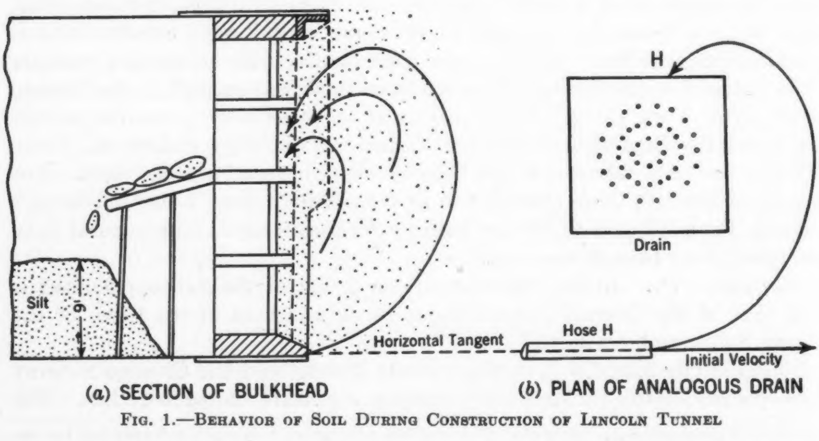
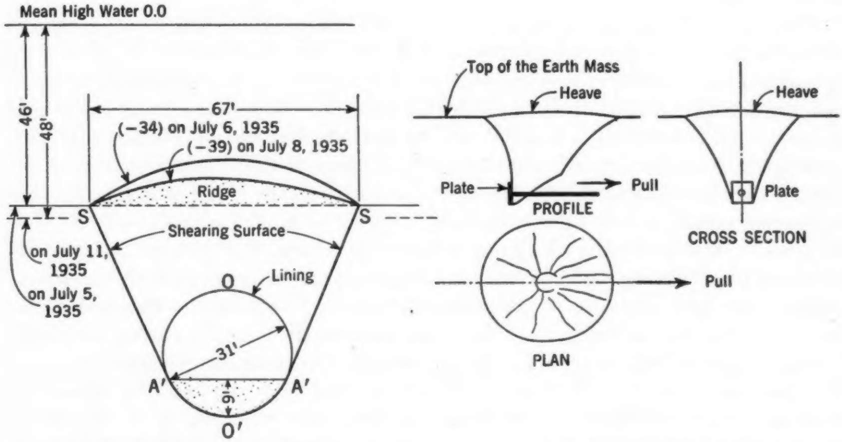


Fig. 1.—Behavior of Soil During Construction of Lincoln Tunnel

shearing strength of the material is overcome, and the bulkhead keeps moving forward. Shearing surfaces are developed in the earth material in front of the bulkhead, both below and above the top of its front face. The particles from the lower part of the mass thus disturbed move upward, follow smooth

curvilinear shearing surfaces, and enter the openings in the front face of the bulkhead approximately as shown in Fig. 1(*a*). The particles from the upper part of that disturbed mass push up the earth above them, creating bulges at the boundary of the earth mass similar to the one shown in Fig. 2(*a*). In the



(*a*) LINCOLN TUNNEL EXPERIENCE (*b*) LABORATORY ANALOGY

Fig. 2.—Material Displaced by Tunnel Driving

Hudson River above the Lincoln Tunnel the advancing shield formed a bulge about 12 ft high. The Chicago Subway is constructed in soft clay, the roof of the tunnel being located beneath a stiff clay crust which separates the soft clay from the superimposed stratum of sand and silt. The heaving of the street was rather slight, because the stiff crust was not broken. The width of this bulge was about twice the diameter of the tunnel. Silt entering through the openings in the bulkhead of the Lincoln Tunnel filled the lower 9 ft of the circular section of the lining which constitutes about 30% of the volume of the displaced silt. Assuming that the remaining 70% formed a parabolic ridge 12 ft high, its width should have been about 67 ft, again practically twice the diameter of the tunnel. Obviously, in each particular case the width of the ridge depends on the depth of the tunnel, other circumstances being equal. The foregoing can be further clarified by analogy: (1) For the movement of particles pushed toward the openings in the bulkhead front face; and (2) for the movement of particles pushed upward and forming the bulge.

Analogy 1.—In the depressed center of the concrete floor of the writer's laboratory there is a drain for emptying water from a tank, through hose H, Fig. 1(*b*), into a sewer. Water leaving hose H follows a path C, Fig. 1(*b*), and enters the drain from the opposite side (point H). The curve in Fig. 1(*b*) is similar in shape to the curve in Fig. 1(*a*). In describing the shield tunnels of the Chicago Subway,[4] Karl Terzaghi, M. Am. Soc. C. E., states that "when watching the clay coming out of the openings, one got quite decidedly the impression that the clay flowed toward the openings from above." Obviously,

the same phenomenon should be expected in the construction of the Lincoln Tunnel.

Analogy 2.—A vertical plate, provided with a horizontal rod and buried in earth, if pulled by the rod, produces both bulging and cracking at the earth surface (Fig. 2(*b*)).

Downward Motion of the Ridge and of the Silt Below It; Friction and Shear.—Formation of the two shearing surfaces A′S, Fig. 2(*a*), divides the silt mass into two individual masses: (a) Mass SA′O′A′S bounded by the shearing surfaces and the lining, and (*b*) the remainder of the silt. When the mass SA′OA′S moves up, its movement is resisted first by the shearing strength of the silt, and subsequently, as the latter is overcome, by friction directed downward (that is, against the direction of the movement). As soon as the ridge or bulge reaches its highest position, a downward motion begins due to the consolidation of the silt mass. The friction is now reversed and acts upward. As time passes, the silt along the shearing surfaces creates a healing process which restores the bond between the two masses. The restoration of bond probably is incomplete in the sense that the disturbed silt does not recover its initial shearing strength during the consolidation period. In this period of adjustment friction opposing the downward motion of mass SA′OA′S is gradually replaced by shearing stresses. After all, friction and shearing stress are phenomena of the same order. If two bodies in contact tend to move in opposite directions, the force opposing the motion is friction; and, if a part of a body or of a mass tends to move with respect to the other part of the same body or mass, the motion is opposed by shearing stress.

For the sake of simplicity, shearing stresses will be assumed acting vertically, and not obliquely, which does not change the reasoning and the results. It is convenient to remember that shearing stresses acting along two mutually perpendicular planes are equal. In other words, existence of a vertical shearing stress, at a point of the mass, implies the existence at the same point of a horizontal shearing stress of the same magnitude.

Consistency of the Remolded Silt Material.—The silt material above the lining and in front of the bulkhead is remolded. As a first approximation, it will be assumed that it acts as a perfect fluid. This is the assumption made by G. M. Rapp and A. H. Baker, Assoc. Members, Am. Soc. C. E., in their report on the Lincoln Tunnel.[2] It implies that at a point of the surface of the lining there is a normal pressure and that both the vertical and horizontal pressures at the same point are numerically equal to that normal pressure. Messrs. Rapp and Baker state[2] that the tangential pressures along the periphery of the lining "were generally less than 2 lbs. per sq. in., and a large portion of them ranged between 0 and 0.5 [lbs. per sq. in.]." In other words, the measured tangential pressures in the silt surrounding the lining were negligible.

Material at the Lower Part of the Lining.—It follows from a simple visual inspection of Fig. 2(*a*) that at the upper part of the lining (above points A′) the moving bulkhead was remolding the silt material and pushing it forward against the passive resistance ("passive pressure") of the silt. In the lower part of the lining (below points A′) the moving bulkhead was making its way through

silt which pressed on it from both sides. This is the "active pressure" of the silt on the lining.

It follows furthermore from Fig. 2(*a*) that the moving bulkhead formed a trough in the silt bounded by the line A′O′A′ (Fig. 2(*a*)). The material within this trough was remolded and, as stated, with a certain approximation may be considered as a liquid. This statement concerns a thin cushion of silt along the lower part of the lining A′O′A′. Conventionally and for the sake of brevity, this part A′O′A′ will be termed "bottom of the lining" in this paper.

Recapitulation.—The foregoing statements concerning the behavior of the silt material around the tunnel may be summarized as follows: The bulkhead pushes the silt forward. A part of it travels paths along curved surfaces and enters the openings in the shield. The remainder bulges upward to form a ridge as the shield driving continues. At the same time, the lower part of the lining makes its way through undisturbed silt. During this time a thin cushion of remolded silt is being formed in the trough A′O′A′ as a result of shoving. The lining, disengaged from undisturbed silt, "floats" in the liquid mud of the "trough" A′O′A′ (Fig. 2(*a*)).

External Forces Acting on the Lining and on the Adjacent Silt Mass

Weight of the Overburden.—During the construction of the Lincoln Tunnel the following facts were recorded:[2] Before driving, the elevation of the river bottom next to the test station was − 46, the datum plane being the mean high-water level in the Hudson River (Fig. 2(*a*)). On July 6, 1935, a ridge 12 ft high was produced by the moving bulkhead (elevation of the top of the ridge, − 34). Due to erosion and other circumstances, this ridge was 7 ft high on July 8 (elevation of the top, − 39). On July 11 the bottom of the river was dredged to elevation − 48.

On the basis of this information[2] (assuming the weight of water in the Hudson River at 64 lb per cu ft and the weight of saturated silt at 104 lb per cu ft), the weight of the overburden on July 8 and on July 11 and afterward is given in Figs. 3(*b*) and 3(*c*), in which the unit weight of the overburden, in pounds per square inch of the horizontal projection of the lining, is shown.

Pressure Records and Standard Loading.—A special silt pressure gage and plugs were designed to measure both normal and tangential pressures on the periphery of the Lincoln Tunnel at a special test station. Accuracy of measurements has been reported[2] to be within 1 lb per sq in. or 2 lb per sq in. The tangential pressures were generally less than 2 lb per sq in., and a large proportion of them ranged between 0 and 0.5 lb per sq in. as already stated. In this paper the tangential pressures at the periphery of the lining of the Lincoln Tunnel are neglected altogether. For comparison, the writer decided to reduce all record data to a standard loading as follows:

(*a*) The lining is loaded with the overburden as of July 11, 1935, the only exception being made for the record of July 8, 1935, in order to discount the effect of the ridge (see heading, "Weight of the Overburden");

(*b*) The total weight of the lining is 18 kips and the total weight of the inside silt deposit is 16 kips per running foot of the tunnel;

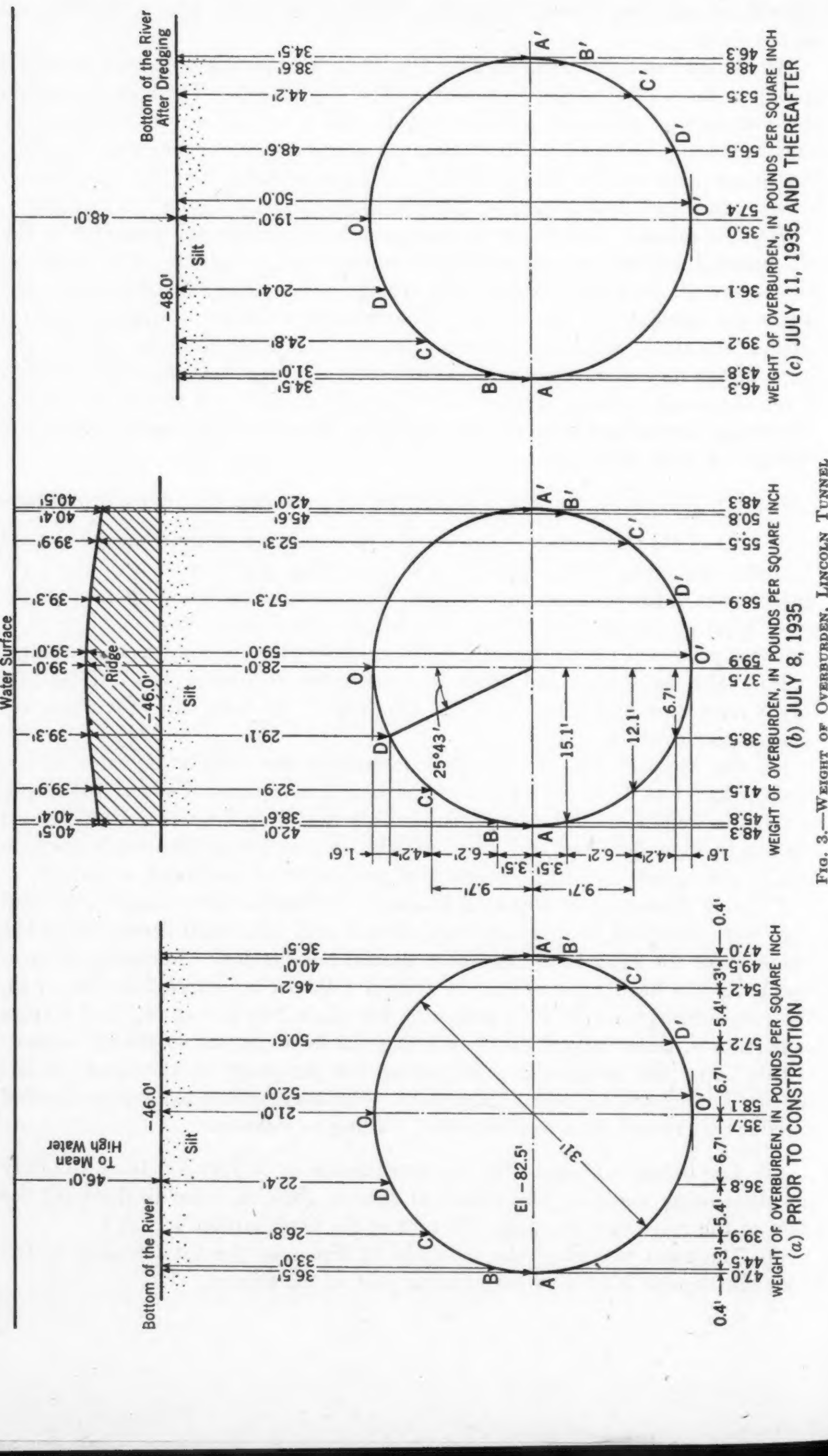


Fig. 3.—Weight of Overburden, Lincoln Tunnel

(*c*) There is no compressed air within the lining above the normal atmospheric pressure. If the situation is different from that described, corrections are introduced. First, the corrections for the compressed air will be computed.

Compressed air was in contact with clay through the openings in the shield only. Hence the expanding lining exerted active pressure on the surrounding silt. As a result, the silt exerted a numerically equal passive resistance, which must be deducted from the measured normal pressure (negative correction). In the case of the Detroit Tunnel, the pressure of the compressed air was transmitted through the pore moisture of clay at the front face and around the lining. This neutral stress decreased the measured pressure (positive correction).

Points on the periphery of the lining of the Lincoln Tunnel where pressure records are available are shown in Fig. 3. To identify different vertical sections of the lining, letters A, B, C, D, and O will be used for the upper part of the lining, whereas letters A′, B′, C′, D′, and O′ correspond to the lower part of the lining (Fig. 3(*b*)). Numerical values of pressures, their averages separately for the upper and the lower semicircumference of the lining, and the compressed air pressures are given in Table 1.

TABLE 1.—RECORD OF PRESSURE INTENSITIES, LINCOLN TUNNEL (LB PER SQ IN.)

Date[a]	Downward Pressure p_d at:[b]				Average p_d	Upward Pressure p_u at:[b]				Average p_u	Compressed air pressure
	Point O	Point D	Point C	Point B		Point B′	Point C′	Point D′	Point O′		
July 8, 1935:											
Recorded	41.5	40.5	43.0	45.5	42.6	48.0	47.5	46.5	46.5	47.1	16
Corrected	39.1	38.1	40.6	43.1	40.2	45.6	45.1	44.1	44.1	44.7	
July 16, 1935:											
Recorded	37.0	36.5	38.0	42.0	38.4	43.5	43.0	42.6	42.7	43.0	16
Corrected	34.6	34.1	35.6	39.6	36.0	41.1	40.6	40.2	40.3	40.6	
Aug. 8, 1935:											
Recorded	38.5	38.0	41.0	43.5	40.3	45.0	44.5	44.0	45.0	44.6	28
Corrected	34.2	33.8	36.8	39.3	36.1	40.8	40.3	39.8	40.8	40 4	
Aug. 29, 1935:											
Recorded	35.0	34.0	37.0	38.0	36.0	40.5	40.0	39.5	40.0	40.0	0
Corrected	....	....	....	....	....	....	....	....	....	....	

[a] "Recorded" values are the original pressures, not corrected for the compressed air pressures; "corrected" values are so corrected. [b] See Fig. 3.

The recorded pressure was corrected by dividing the difference of the average recorded pressures on two given days by the corresponding difference of compressed air pressures. The increase in compressed air pressure from 16 lb per sq in., on July 16, to 28 lb per sq in., on August 8, corresponds to an increase in recorded pressure on the upper part of the lining of $\frac{40.3 - 38.4}{12} = 0.16$ lb per sq in. per pound of compressed air pressure. The corresponding value for the lower part of the lining is $\frac{44.6 - 43.0}{12} = 0.13$ lb per sq in. The reduction in compressed air pressure from 28 lb per sq in., on August 8, to zero, on August 29, is computed as 0.16 lb per sq in. for both parts of the lining.

In view of these considerations all recorded pressures were corrected by deducting from them as an average, 0.15 lb per sq in. per pound of compressed air pressure on that day.

Analogy 3.—Before discussing the play of forces acting on both the lining and the adjacent silt mass, consider the following analogy:

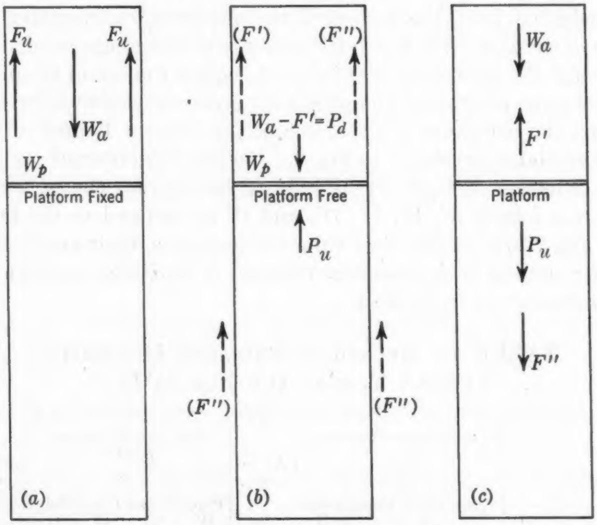


FIG. 4.—ELEVATOR SHAFT ANALOGY

In Fig. 4, a perfectly rigid platform is fixed at a certain position in a hypothetical elevator shaft of unit cross-section area. It supports its own weight W_p and the weight W_a of nonconsolidated silt in the shaft above the platform. Below the platform the shaft is completely filled with a perfectly consolidated silt. As the silt above the platform consolidates, friction forces F_u develop along the walls of the shaft, relieving the load on the platform to that extent. In other words (see Fig. 4(*a*)), the resultant downward force acting on the platform, except its weight, is

$$P_d = W_a - F_u \qquad (1)$$

and since the platform is wedged in position it reacts upward in the same degree in the silt load. Forces acting upward are distributed throughout the upper silt mass. The resultant of these forces is equal numerically to F_u. Since the platform is fixed (by brakes, say, or wedges), it does not exert any pressure on the silt below it; and since the latter is fully consolidated there is no movement and no friction below the platform.

If the platform is released, a consolidation process begins beneath it, and then the total force or load moving the platform downward is $P_d + W_p$. The platform moves downward following the consolidating silt mass; and this new movement is opposed by some new friction forces F' and F'' on the walls of

the shaft and by the reaction of the mass P_u at the bottom of the platform (Fig. 4(*b*)). The weight of the mass below the platform has already caused complete consolidation of the material and need not be considered. Consequently, the mass under the platform should be assumed as being acted upon by a force P_u and vertical forces spread throughout the mass and having F'' for the resultant (Fig. 4(*b*)). The final distribution of the forces acting on the silt mass both above and below the platform is shown in Fig. 4(*c*).

Since the platform and the silt, on which it reposes, move, forces of inertia should be taken into consideration. The possible motion is very slow, however, and the forces of inertia will be disregarded.

Forces Acting on the Silt Mass Above and Below the Lining.—Analogy 3, as illustrated by Fig. 4, can be applied to the example in Fig. 5, which shows the distribution of forces acting on the lining of the Lincoln Tunnel during its construction. As in Fig. 4, W_a is the weight of the overburden above the lining or "platform" (see Fig. 3); W_p is the total weight of the lining and the inside silt deposit (that is, silt which entered the lining as in Fig. 1(*a*)), P_d and P_u are pressures measured during the construction (see Table 1). The total value of the weight W_p is 34 kips per running foot of the tunnel, 18 kips being the weight of the lining itself, and 16 kips being the weight of the inside silt deposit. The distribution of the weight W_p along the surface of contact with the silt mass at the bottom of the lining A'O'A', in Fig. 2(*a*), is unknown. Two assumptions will be made: (*a*) The lining is perfectly rigid, and the weight W_p is uniformly distributed along the horizontal projection of the lining; and (*b*) the lining is perfectly nonrigid, and the weights of the lining and of the inside silt deposit are distributed along the horizontal projections of the lining proportionately to the respective thicknesses of the lining and of the inside silt deposit in a given vertical section of the tunnel. The results are shown in Fig. 6. The actual situation is between these two assumptions. It is to be noted also that, in the case of the lining (Fig. 5), all "friction forces," as shown in Fig. 4, are "shearing forces."

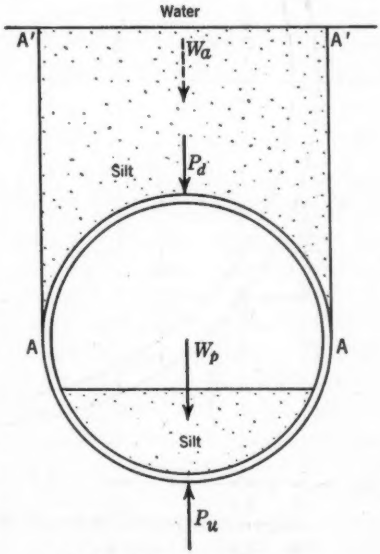


FIG. 5.—VERTICAL FORCES ACTING ON THE LINING

Difference Between the Analogy (Fig. 4) and the Case of the Actual Lining.—In Fig. 4, the clay mass under the platform is assumed fully consolidated, and as soon as the "platform" is released, a new process of additional consolidation begins under the weight of the platform and the silt deposit above it. In the case of the lining of the Lincoln Tunnel the situation was different: First, in shoving the tunnel a cushion of silt along the bottom of the lining was remolded

and became liquid; and, second, the overburden under which the silt along the bottom of the lining was consolidated was not increased, but was relieved as the lining passed through it. This can be disclosed by comparing the weight of the overburden at points O′, D′, C′, B′ (Fig. 3) and the values of $P_d + W_p$ (Table 1 and Fig. 6).

Interpretation of the Curve $W_a - P_d$.—In Fig. 7 the values of the weight of the overburden W_a and of the downward pressure P_d are plotted and their difference $W_a - P_d$ is represented in the form of curves. The difference $W_a - P_d$ is the relieved weight of the overburden at a given vertical section of the lining, as expressed in pounds per square inch on the horizontal projection of the lining. In computing these values, the difference $W_a - P_d$ at the center line of the lining (point O) was assumed to be zero.

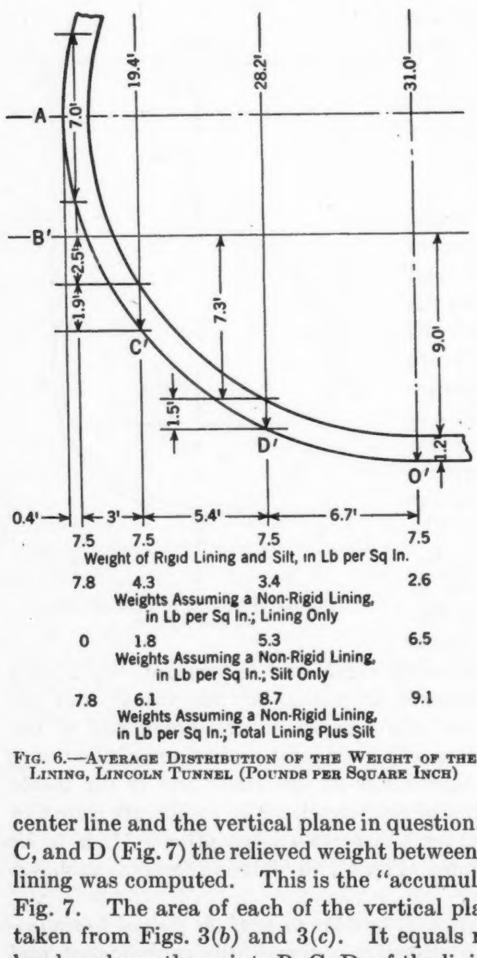


FIG. 6.—AVERAGE DISTRIBUTION OF THE WEIGHT OF THE LINING, LINCOLN TUNNEL (POUNDS PER SQUARE INCH)

Due to symmetry, there can be no horizontal shear and hence no vertical shear at point O (center line of the lining). In other words, there can be no relief in the weight of the overburden at that point. Another feature of the curve $(W_a - P_d)$, and of the curves in Fig. 8, is the horizontal tangent at the center line of the lining, obviously due to symmetry.

Conditions of equilibrium require that at any vertical plane between the center line of the lining and its sides (haunches), the total shearing force, acting upward at that plane, must balance the relieved weight between that center line and the vertical plane in question. For each of the vertical planes B, C, and D (Fig. 7) the relieved weight between that plane and the center line of the lining was computed. This is the "accumulated relieved weight in pounds," in Fig. 7. The area of each of the vertical planes B, C, D, above the lining was taken from Figs. 3(*b*) and 3(*c*). It equals numerically the height of the overburden above the points B, C, D, of the lining. The corresponding figures are to be found at the line "area of action of the shearing stress in square feet" (Fig. 7). By dividing the accumulated relieved weight at each plane by the area of the latter, the average shearing stresses (in pounds per square foot) were computed. At no place is this shearing stress more than 156 lb per sq ft. The

value of the shearing strength of the Hudson River silt may be estimated from comparison with other similar materials of the Atlantic Coast. For instance, the shearing strength of the silt in the Thames River at New London, Conn., which is more wet and hence less resistant than the Hudson River silt, is about 150 lb per sq ft as determined by direct shear test. Apparently the average shearing stress above the lining of the Lincoln tunnel reached the value close to its shearing strength; but no data exist for judging what was the maximum value of that shearing stress. The ordinates of the curves of the "average upward shearing stress" beyond the sides (haunches) of the lining must decrease and must vanish at a certain distance from the tunnel.

Comparison of the Vertical Upward and Downward Forces Acting on the Lining.—These forces are: The downward pressure P_d; the weight of the lining including the inside silt deposit (W_p in its entirety); and the upward pressure P_u. Their resultant,

$$F_d = P_d + W_p - P_u \qquad (2)$$

is plotted in Fig. 8. The values of W_p are from Fig. 6, for rigid and nonrigid linings, respectively. Since the force $P_d + W_p$ acting downward must be balanced, and the force P_u is not large enough for this purpose, the area bounded by curve F_d, representing the difference of the downward and upward vertical forces acting on the lining shows the force balancing the system in question.

This force may be interpreted as: (*a*) A downward shearing force (such as force F'' in Fig. 4(*c*)); (*b*) a force tending to make the lining settle; or (*c*) items (*a*) and (*b*) combined. There are no data for estimating the value of the downward shearing forces; and some assumption should be made, as, for instance, that the upward and the downward shearing forces at each vertical plane are numerically equal. For comparison purposes, curve F_u has been plotted in Fig. 8. Under the approximate assumptions made, the difference between areas bounded by curves F_d and F_u corresponds to the value of the force that tends to make the tunnel settle. On July 8, 1935, this force ($F_d - F_u$) was considerable (Fig. 8(*a*)), due to the presence of the ridge of freshly remolded silt material. It reduced practically to zero after dredging (July 16, 1935), and afterward gradually increased until August 29, 1935.

Horizontal Pressure on the Lining.—To plot the horizontal pressure on the lining, Table 1 was used again. In Fig. 9 the recorded pressures corrected for compressed air are plotted horizontally at the points of measurement shown in Fig. 3.

Due to the pressure of the surcharge material forming the ridge, the ordinates of curve 1 are about 4 lb per sq in. larger than the ordinates of curves 2, 3, and 4, which are practically the same. The horizontal pressure at the sides (haunches) (point A) on August 29, 1935, was about 40 lb per sq in. Deducting from this value 21.3 lb per sq in. corresponding to the pressure of water on the bottom of the Hudson River, the difference would be 40.0 − 21.3 = 18.7 lb per sq in. The vertical pressure at the same point and on the same date was 46.3 − 21.3 = 25.0 lb per sq in. (points A and A′, Fig. 3(*b*)), which corresponds to a ratio of $\frac{18.7}{25} = 0.74$. In a perfect liquid this ratio would

be 1. Using the value of 0.74 in the Rankine formula, the corresponding angle of friction of the silt material on August 29, 1935, was about 9°, which is a reasonable value for that kind of material.

The well-known Rankine formula was derived by its author as applying to undisturbed earth mass in which the unit pressure increases from the top downward, following a straight-line law (triangular pressure distribution).

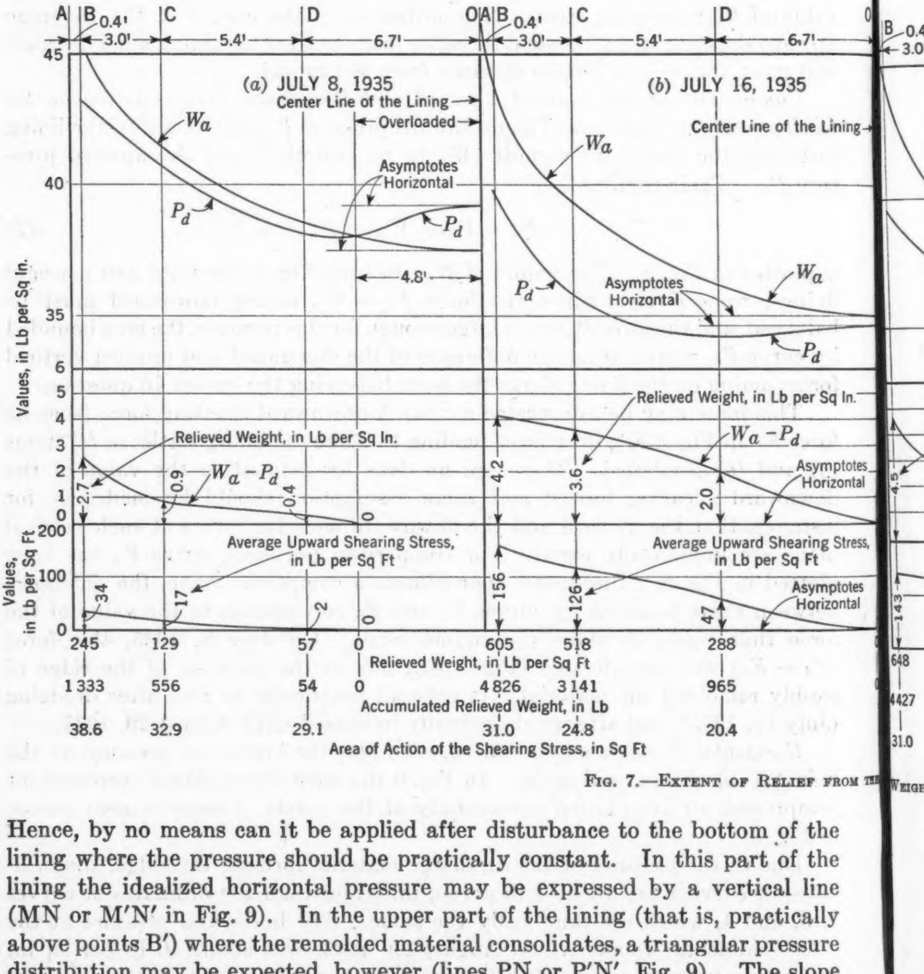
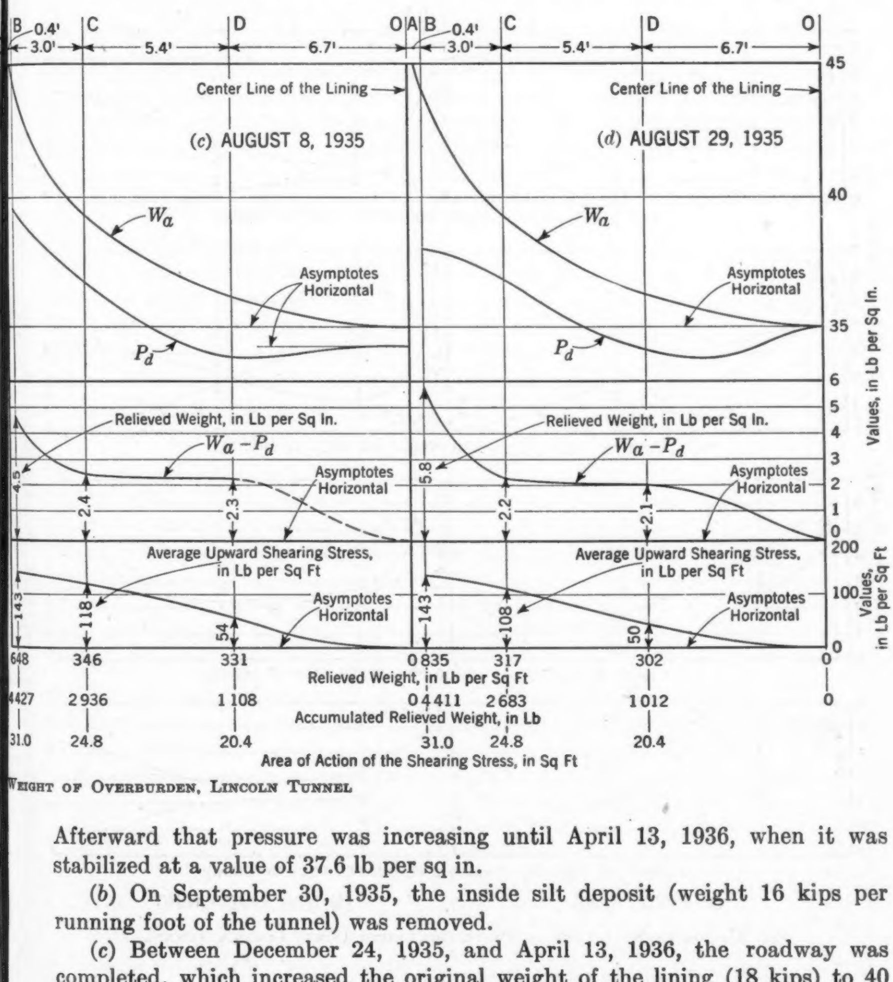


FIG. 7.—EXTENT OF RELIEF FROM THE WEIGHT

Hence, by no means can it be applied after disturbance to the bottom of the lining where the pressure should be practically constant. In this part of the lining the idealized horizontal pressure may be expressed by a vertical line (MN or M′N′ in Fig. 9). In the upper part of the lining (that is, practically above points B′) where the remolded material consolidates, a triangular pressure distribution may be expected, however (lines PN or P′N′, Fig. 9). The slope of the lines PN, P′N′ is too steep due to the fact that the curves, shown in Fig. 9, are constructed considering the remolded silt as a perfect liquid. In reality the pressure at point A may be correct whereas that at point O is exaggerated. Point P″, Fig. 9, is a corrected position of point P′ (for an "angle of friction" of silt equal to 9°).

Change in Average Pressure on the Lining after August 29, 1935.—Detailed data concerning the pressure distribution on the lining after August 29, 1935, are not available. The curve showing the average pressure on the lining[2] reveals the following facts:

(*a*) The average pressure was decreasing until December 24, 1935, when it reached its minimum (36 lb per sq in.). On that day the tie rods were erected.



WEIGHT OF OVERBURDEN, LINCOLN TUNNEL

Afterward that pressure was increasing until April 13, 1936, when it was stabilized at a value of 37.6 lb per sq in.

(*b*) On September 30, 1935, the inside silt deposit (weight 16 kips per running foot of the tunnel) was removed.

(*c*) Between December 24, 1935, and April 13, 1936, the roadway was completed, which increased the original weight of the lining (18 kips) to 40 kips per running foot of the tunnel.

A change in load of 1,000 lb inside the lining causes an average change in pressure on the lining equal to $\frac{1,000}{2 \times 31 \times 144} = 0.11$ lb per sq in. Corrections due to such changes are:

(1) On December 24, 1935, for the inside silt deposit, $0.11 \times 16 = 1.8$ lb per sq in., the corrected pressure being equal to $36.0 + 1.8 = 37.8$ lb per sq in.

(2) On April 13, 1936, corrections are indicated for both the inside silt deposit and the additional weight of the roadway, these two corrections being

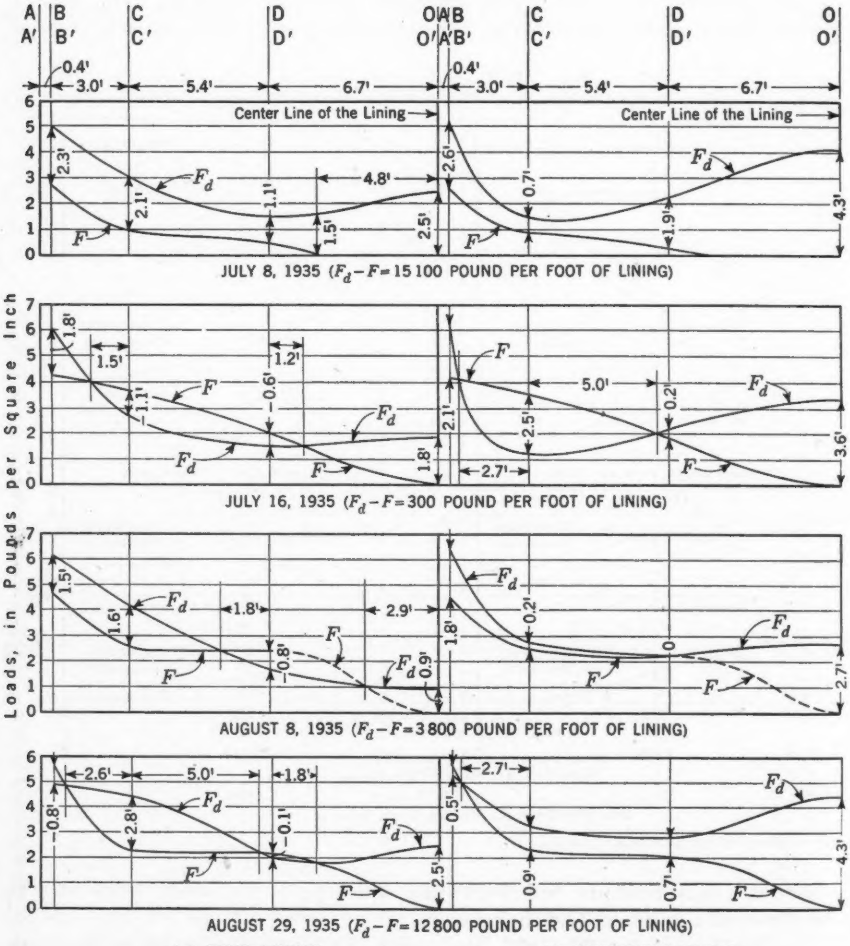


FIG. 8.—FORCES THAT TEND TO DRIVE THE LINING DOWN, LINCOLN TUNNEL

of opposite signs. The final pressure, 37.6 lb per sq in., is decreased by $0.11 \times (22 - 16) = 0.7$ lb per sq in., yielding a value of 36.9 lb per sq in.

These results are plotted in Fig. 10, in which the solid line represents the average pressure on the lining as measured in the field, and the dotted line shows that pressure as reduced to the specified standard loading. The reduced

pressure decreased systematically until April 13, 1936, when the field records reported a stabilization of the average pressure on the lining.

The published material obtained during the construction of the Lincoln Tunnel[2] is not detailed enough and covers a rather short period (253 days). As an example of a more complete procedure, the Detroit Tunnel[3] may be cited. Pressure observations at that tunnel covered the period from December, 1930, to January, 1941—about eleven years. In the ordinary case such a long period of observations may be unnecessary; but it seems advisable to observe pressures on any tunnel during a reasonably long-time interval, which may be estimated, for instance, at three years; this may involve but a small additional expense. It was found at the Detroit Tunnel that the vertical pressure increased and tended to return to its original (preconstruction) value, whereas the horizontal remained more or less constant during the entire period of observations. These facts may be explained by a partial or complete disappearance of shearing stresses, both vertical and horizontal. No definite conclusions as to the disappearance of the shearing stresses can be drawn on the basis of published reports on the Lincoln Tunnel.[2]

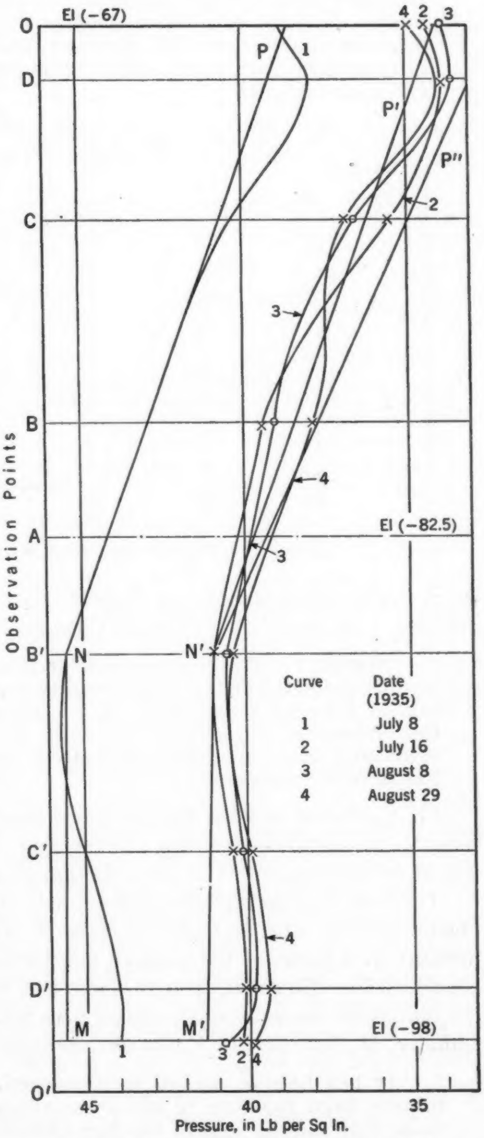


Fig. 9.—Horizontal Pressure on the Lincoln Tunnel Lining

Decrease in the Horizontal Diameter of the Lincoln Tunnel

According to an official statement:[2]

"Measurements of the horizontal and vertical diameters of the lining showed distortions similar to those that have been observed in other tunnels in Hudson River silt. Immediately after erection the hori-

zontal diameter began to shorten, and the vertical diameter began to lengthen by about the same amount. Nine days after the lining had been erected at the test station, the horizontal diameter had decreased and the vertical diameter had increased, each, by $1\frac{1}{2}$ inches. This was the maximum distortion observed at the test station. From then on, the horizontal diameter increased and the vertical diameter decreased until they returned to their original values six months after the lining was erected [about December 24, 1935; see Fig. 10]."

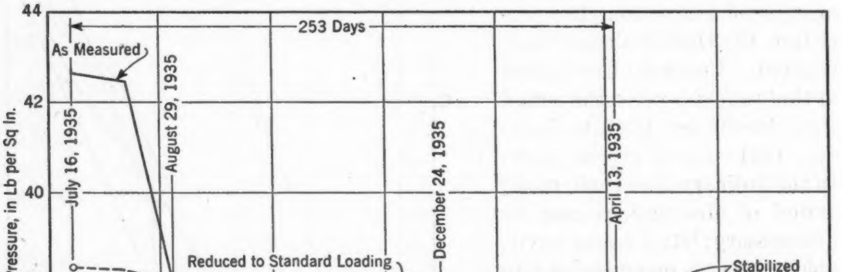


FIG. 10.—AVERAGE PRESSURE ON THE LINING, LINCOLN TUNNEL

A similar phenomenon was observed during the construction of the Chicago Subway. As stated by Professor Terzaghi:[4]

"Within the first few days after the rings were assembled, the (horizontal) diameter of the rings steadily decreased. The greatest shortening ranged between 1 and $1\frac{1}{2}$ in. * * * After the initial period of shortening the horizontal diameter of the tube increased and after about one year assumed a constant value, which was about $1\frac{1}{2}$ in. in excess of the initial length of the diameter."

Elongation of vertical diameter and shortening of the horizontal diameter seem to be characteristic also of tunneling in North River (New York City) silt where the material is so unstable that the shield can be shoved "blind."

The foregoing examples include tunnels constructed by the shield method in plastic organic silts and plastic clays in which the ratio of the horizontal pressure at a point of the mass to the vertical pressure at the same point is considerable. However, in soils where this ratio is small, such as sandy soils, the horizontal diameter of the lining does not shorten. For instance, John H. Quimby, M. Am. Soc. C. E., in correspondence with the writer, stated:

"It has been a maxim in constructing all of the East River shield tunnels from the time of the Pennsylvania R. R. tunnels to the present time, that the top iron in the ring should be kept high, i.e. that each ring should be constructed 'out of round' with the vertical diameter longer than the horizontal diameter, since it is practically the universal experience in the soils under the East River, and under the Harlem River as well, that the tunnel rings tend to settle down and elongate on the horizontal diameter."

It follows from this evidence that the shortening of the horizontal diameter of the lining is not characteristic of the shield method generally as is sometimes believed, but that it occurs in plastic soils only, in which the ratio of the horizontal pressure to the vertical is considerable.

It seems undeniable that, to shorten the horizontal diameter, the total horizontal force acting on the lining must be larger than the vertical; yet in the case of the Lincoln Tunnel a careful computation of these forces for all pressure records available since July 6, 1935, did not reveal this to be the case. It must be concluded therefore that the cause of the shortening is to be sought at the very beginning of the shoving. The following explanation may be offered. Before applying force to the shield any point of the bottom of the lining A′O′A′ (Fig. 2) is under the action of a vertical pressure caused by the overburden (silt and water in the river, Fig. 3(*a*)). The bulkhead relieves this vertical pressure (see heading, "Difference between the Analogy (Fig. 4) and the Case of the Actual Lining"); but the lateral pressure that exists in the natural silt mass still persists at the points A′, B′, C′, and D′ and squeezes the lower part of the lining, thus shortening the horizontal diameter. Since the lining is rigid the compressive force at the shield acts not only on the head section of the lining, but also on rear sections through a certain distance. This explains why the decrease in horizontal diameter lasts some days after the lining is set.

So far as its shape is concerned, the graph of lateral pressure distribution at the initial moment of shoving should be similar to line P′N′ in Fig. 9 and its continuation (Rankine or triangular pressure distribution). Due to the remolding of the silt in the bottom of the lining, this pressure is partly replaced by a uniform pressure (distributed approximately as shown by line N′M′ in Fig. 9). There is an opinion that the use of the Rankine formula in the design and analysis of tunnels should be discontinued. Evidence presented in this paper makes it advisable to think twice before doing so.

Action of Compressed Air

The downward vertical pressure P_d, as shown in the first line corresponding to each date in Table 1, may be separated into two parts: (*a*) The active pressure that was exerted by the overburden on the lining as shown in the second line corresponding to each date in Table 1, and (*b*) the pressure exerted by the lining on the overburden due to the action of compressed air during construction. This is the passive pressure of the overburden on the lining, and the pressure-measuring devices recorded the sum of the two. Of course, at no time was the active pressure of the overburden on the lining larger than the weight of the overburden (Fig. 3). Consider, for instance, the action of the compressed air on August 8, 1935. At an air pressure of 28 lb per sq in., the compression stress in the lining at its inside periphery was also 28 lb per sq in. At the outside periphery the average compression stress was only $0.15 \times 28 = 4.2$ lb per sq in., which means that the overburden was loaded approximately with average unit loads of that value acting upward. The resultant of these loads per running foot on half of the lining was $4.2 \times 144 \times 15.5 = 9{,}374$ lb. Upon dividing this value by the area of action of the shearing stress at the sides (haunches)

of the lining (34.5 sq ft, see Fig. 3(*c*)), the average value of the downward shearing stress at the haunches caused by the compressed air was $\frac{9,374}{34.5} = 272$ lb per sq in. The shearing stresses in the silt mass caused by the weight of the overburden (Fig. 7) and those due to the action of the compressed air acted in opposite directions. Therefore, the compressed air not only restored the original preconstruction condition in the silt, but even created some excess of safety.

Conclusions

1. During the construction of the Lincoln Tunnel, the weight of the overburden above the tube was relieved by the upward vertical shearing stresses.

2. There were downward vertical shearing stresses below the lining of the Lincoln Tunnel.

3. A diagram of shearing stresses for different points of the periphery, plotted from a horizontal line, has a zero ordinate and a horizontal tangent at the center line of the lining. The ordinates of this diagram increase toward the sides (haunches) of the lining. Such diagrams should be extended beyond the sides (haunches) in order to satisfy conditions of equilibrium; but there are no available data as to the shape of such extensions.

4. The average normal pressure on the lining of the Lincoln Tunnel as reduced to the conditions of a standard loading decreased steadily until it apparently assumed a constant value about eight months after the erection of the lining.

5. The decrease in the average pressure on the lining of the Lincoln Tunnel as cited in conclusion 4 should indicate the gradual disappearance of the shearing stresses acting on the mass. No definite conclusions on this subject can be drawn from the information published to date.

6. The decrease in horizontal diameter of the lining of the Lincoln Tunnel is a phenomenon that has been observed in a number of other cases in which the shield method was applied in plastic soils, such as organic silts or plastic clays, but not in sands or similar soils. It is caused apparently by the action of a considerable lateral pressure in the lower part of the lining at the initial stage of shoving, which action is combined with a simultaneous relief in vertical pressure due to the passage of the shield.

7. Diagrams of horizontal pressure on the lining of the Lincoln Tunnel do not sustain the belief that the Rankine formula must be abandoned in the design and analysis of tunnels.

Acknowledgment

The writer extends his thanks to the New York City Tunnel Authority, the Port of New York Authority, and to John H. Quimby, M. Am. Soc. C. E., and G. M. Rapp, Assoc. M. Am. Soc. C. E., for supplying and collating valuable unpublished data used in preparing this paper. James H. Dugan, M. Am. Soc. C. E., was very helpful to the writer in advising him on tunnel terminology.

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PAPERS

CONSTRUCTION OF THE FLOW NET FOR HYDRAULIC DESIGN

BY H. ALDEN FOSTER,[1] M. AM. SOC. C. E.

SYNOPSIS

The general principle of the flow net has been known for many years, and its application to problems in hydraulics, electricity, and heat transmission has been described in various publications. The construction of a flow net by customary methods, however, is very tedious and the results are frequently far from precise.

In this paper, certain graphical and analytical methods are presented by means of which the construction of the hydraulic flow net in any particular case can be facilitated greatly, and much greater precision can be attained in the determination of velocities and pressures. The methods are applied both to conditions of fixed boundaries and to cases in which one of the boundaries is exposed to the atmosphere. Their application to certain cases of three-dimensional flow is also explained, together with a specific example.

INTRODUCTION

The usefulness of the flow net in hydraulic design has long been recognized, and graphical methods for constructing such a net have been described (1).[2] The flow net is of particular importance in any design problem involving the determination of pressures occurring under curvilinear flow. Under such conditions, the pressure variation does not follow the hydrostatic law (that pressure increases directly in proportion to depth of water), since the velocity is not uniform across the section. To compute the pressure by Bernoulli's formula, the velocity at any point must be known, and in many problems the only way in which the velocity can be determined with any degree of precision is by a model test or by construction of a flow net. If a flow net can be constructed accurately in conformance with certain fundamental conditions, the

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **October 1, 1944.**

[1] Designing Engr., Parsons, Brinckerhoff, Hogan & Macdonald, New York, N. Y.

[2] Numerals in parentheses, thus: (1), refer to corresponding items in the Bibliography (see Appendix I).

velocities determined from the flow net should conform closely to those in the actual structure, provided friction and impact losses are of a minor nature.

In the flow-net method as generally presented, emphasis is placed on constructing the net by freehand sketching, with gradual adjustment and correction until the streamlines and potential lines intersect at right angles and form curvilinear squares. This is a laborious process; and even when accurately done the resulting velocity determinations will not be very precise if the streamlines are close together, or if the streamlines are only slightly curved and the variation in velocity across the section is rather gradual. A graphical method of checking the spacing of the streamlines, which also permits a more precise determination of the velocities, is described in this paper. The basis of this method is given by Hunter Rouse, M. Am. Soc. C. E. (2), and by R. L. Daugherty (3). However, so far as the writer knows, the detailed application of the method has not been made generally available to engineers.

Notation

The letter symbols used in this paper are defined where they first appear and are assembled for convenience of reference in Appendix II.

Two-Dimensional Flow with Fixed Boundaries

Consider first the case of two-dimensional flow within fixed boundaries, as shown in Fig. 1(*a*). A section of the water passage one unit in depth (normal to the plane of the paper) is considered. The radii of curvature of the fixed boundaries are r_s and r_c; these need not be concentric. The width of the water passage, measured along the curved potential line, APC, is d. Assuming that the streamlines have been plotted, as in Fig. 1(*a*), the radius of curvature of the streamline at any point along the line APC can be shown as a curve of r versus y, line $A_1P_1C_1$ in Fig. 1(*c*), y being measured from the outer curved boundary toward the center of curvature as indicated in Fig. 1(*a*). Then the basic theory of the flow net, with uniform energy distribution along the water passage (no friction or impact losses) requires that

$$\frac{V}{r} = \frac{dV}{dy} \qquad (1)$$

in which V = velocity at any point, P, in the water passage, and r = radius of curvature of the streamline at point P.

A curve, A_2C_2, showing V versus y can be constructed in Fig. 1(*c*) as follows:

A value of unity is assumed for the velocity, V_s, at point A_2, where $y = 0$. Starting at this point, the slope (dV/dy) of the velocity curve will be V_s/r_s, according to Eq. 1. This slope is laid off graphically, as shown on the diagram, by swinging point A_1 along a circular arc of radius r_s, until it reaches the position A_3 on the axis of y. The straight line A_3A_2, extended a short distance beyond A_2 in the direction of y, gives the tangent of the velocity curve at point A_2, in accordance with Eq. 1. For all practical purposes, this piece of tangent may be taken as the true curve for a short distance, such as A_2D_2.

In this way, a new value of V is obtained at point D_2; at which point the process is repeated, using the value of r given by the point D_1 on A_1C_1, directly above point D_2 (construction not indicated in Fig. 1(*c*)). Thus, a new determination

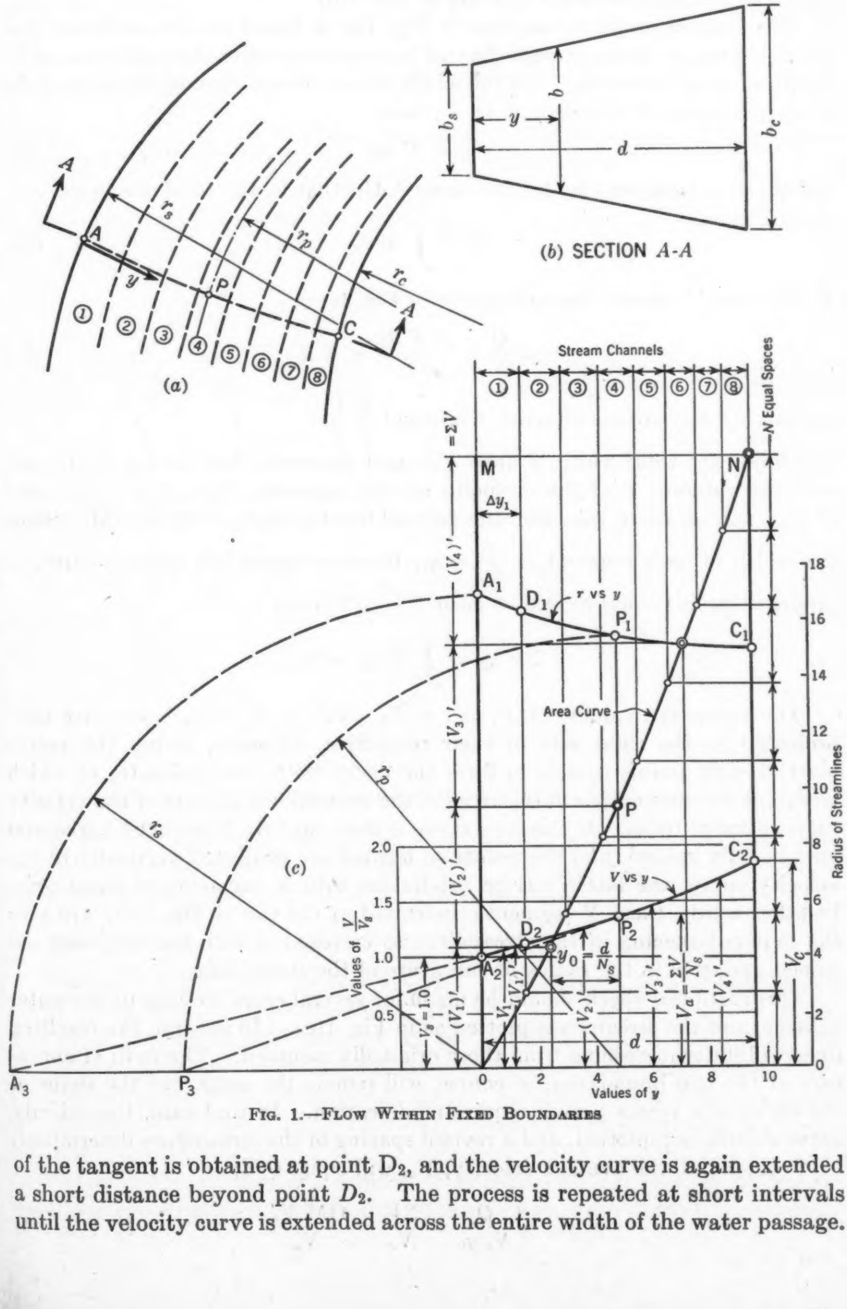

FIG. 1.—FLOW WITHIN FIXED BOUNDARIES

of the tangent is obtained at point D_2, and the velocity curve is again extended a short distance beyond point D_2. The process is repeated at short intervals until the velocity curve is extended across the entire width of the water passage.

The ordinates of the velocity curve, A_2C_2, so determined, give values of V/V_s at any point in the line APC of Fig. 1(*a*). Note that, in this construction, the same scale must be used for r and y in Fig. 1(*c*), but this does not need to be the same as the scale of the flow net in Fig. 1(*a*).

The spacing of the streamlines in Fig. 1(*a*) is based on the condition that the discharge, q, of any stream channel between successive streamlines must be constant for all channels. Let the width of any stream channel be Δy, and the average velocity of the channel be V; then

$$q = V\,\Delta y \qquad (2a)$$

Let Q = the total area under the curve A_2C_2; that is, the total discharge:

$$Q = \int V\,dy \qquad (2b)$$

If there are N stream channels (eight in Fig. 1(*a*)),

$$q = \frac{Q}{N} = \frac{\int V\,dy}{N} = V\,\Delta y \qquad (2c)$$

and Δy, for any stream channel, will equal $\frac{Q}{N}\frac{1}{V}$.

Divide the total width, d, into N_s equal segments (four in Fig. 1(*c*)), and note the value of V at the midpoint of each segment, $(V_1)'$, $(V_2)'$, $(V_3)'$, and $(V_4)'$. Lay off these values on the vertical line through $y = 0$, as OM. Since the width of each segment is $\frac{d}{N_s} = y_0$, the area under the velocity curve is (approximately) equal to $\overline{\text{OM}} \times y_0$ or $\Sigma V\,y_0$; that is,

$$\Sigma V\,y_0 = \int V\,dy = Q \qquad (3)$$

The successive values, $(V_1)'$, $(V_1 + V_2)'$, $(V_1 + V_2 + V_3)'$, etc., are then projected to the right side of their respective segments, giving the points marked with double circles, to form the curve OPN, the ordinates of which represent the summation of the areas of the respective segments of the velocity curve, divided by y_0. If this area curve is then cut into N parts by horizontal lines equally spaced, and the points so located are projected vertically to the velocity curve, the latter will be subdivided into N segments of equal area. In other words, these N segments (indicated at the top of Fig. 1(*c*)) will give the required spacing of the streamlines to correspond with the originally assumed variation in the radius of curvature of the streamlines.

This graphical check should be made for several cross sections of the water passage, and the streamlines plotted as in Fig. 1(*a*). In general, the resulting lines will differ in spacing from those originally assumed. The radii of curvature at the two boundaries, of course, will remain the same, but the shape of the curve of r versus y may require modification. In that case, the velocity curve should be replotted, and a revised spacing of the streamlines determined.

From Eq. 3, the average velocity, V_a, will equal Q/d; or

$$V_a = \frac{Q}{N_s\,y_0} = \frac{\Sigma V}{N_s} = \frac{\overline{\text{OM}}\,V_s}{N_s} \qquad (4)$$

and $V_a/V_s = \overline{\text{OM}}/N_s$. The ratio V_a/V_s can therefore be determined directly from the "area curve," as shown in Fig. 1(*c*). Then,

$$V_s = \frac{V_a}{V_a/V_s} \qquad \text{(5a)}$$

and

$$V_c = V_s \times \frac{V_c}{V_s} \qquad \text{(5b)}$$

The ratio V_c/V_s may be read directly at the right end of the velocity curve, where $y = d$.

Free Vortex.—If the two boundary surfaces are cylindrical and concentric, and all the streamlines have the same center of curvature, $dy = - dr$. From Eq. 1, $\frac{dV}{V} = \frac{dy}{r}$. Hence,

$$\frac{dV}{V} + \frac{dr}{r} = 0 \qquad \text{(6)}$$

Integrating Eq. 6, $\log V + \log r = \log C$, or $\log (V\,r) = \log C$, in which C is a constant. Hence, $V\,r = C = V_s\,r_s$; or

$$V = \frac{V_s\,r_s}{r} \qquad \text{(7)}$$

This is the condition existing in a "free vortex" (2*a*); but

$$Q = \int_0^d V\,dy = -\int_{r_s}^{r_c} \frac{V_s\,r_s\,dr}{r} = V_s\,r_s \times \log_e \frac{r_s}{r_c} \qquad \text{(8)}$$

Since $Q = V_a\,d$,

$$V_a = \frac{V_s\,r_s}{d} \log_e \frac{r_s}{r_c} \qquad \text{(9)}$$

and

$$\frac{V_a}{V_s} = \frac{r_s}{d} \log_e m \qquad \text{(10)}$$

in which $m = \frac{r_s}{r_c}$. Accordingly, with two-dimensional flow and parallel cylindrical walls for the water passage, so that "free-vortex" conditions apply, the velocity distribution can be obtained directly by Eqs. 7 and 10, and the graphical construction previously described can be omitted.

Three-Dimensional Flow with Fixed Boundaries

The basic formula for the flow net (Eq. 1), as derived by Professor Rouse (2*b*)(2*c*), depends on the variation of the velocity in the direction y, taken normal to the streamline tangent at the particular point considered. In three-dimensional flow, this tangent may not remain in the same plane as one progresses along a given streamline in space. Likewise, the direction of y may not remain in the same plane. However, the mathematical relation shown by Eq. 1 will remain valid.

The previous discussion is applied to two-dimensional flow, or where the water passage is rectangular in section—that is, of constant depth b, perpendicu-

lar to the plane of the paper. For three-dimensional flow, or where the depth of the water passage is variable, the same methods can be used to determine the relative velocities, V/V_s, provided that the radius of curvature in space of the streamline at a given point remains approximately equal to what it would be for two-dimensional flow. This will be true if the cross section of the stream passage remains constant in the direction of flow; and, even if there is some degree of convergence in the water passage, the curvature of the streamlines may not change to any extent (3). Experimental proof of the assumption that the two-dimensional methods may be applied directly to three-dimensional flow is noted subsequently in this paper.

Assuming that the velocity distribution obtained by the two-dimensional method will also apply to three-dimensional flow, the actual velocities may be determined as follows:

Trapezoidal Section.—Assume that the cross section of the water passage is a trapezoid, as in Fig. 1(*b*). Divide the water passage into N strips of width Δy; the widths Δy may be equal or variable. Then total discharge is

$$Q = \sum_{y=0}^{y=d} V\, b\, \Delta y = \sum_{y=0}^{y=d} \frac{V}{V_s} V_s\, b\, \Delta y \qquad (11a)$$

If the values of Δy are equal,

$$Q = \Delta y\, V_s \sum \left(\frac{V}{V_s} b \right) \qquad (11b)$$

and

$$V_s = \frac{Q}{\Delta y \sum \left(\frac{V}{V_s} b \right)} \qquad (12)$$

If the values of Δy correspond to the streamlines as obtained for the two-dimensional condition, $V\, \Delta y$ = constant = $V_1\, \Delta y_1$, in which V_1 is the average velocity in stream channel 1, Fig. 1(*a*), and Δy_1 is the width of that channel. Hence,

$$Q = V_1\, \Delta y_1\, \Sigma b \qquad (13)$$

$$V_1 = \frac{Q}{\Delta y_1\, \Sigma b} \qquad (14a)$$

and

$$V_s = \frac{V_1}{V_1/V_s} \qquad (14b)$$

In either case, $V_c = V_s\, (V_c/V_s)$. In using Eqs. 11 to 14, the values of V/V_s, V_c/V_s, and V_1/V_s are taken from the velocity curve of Fig. 1(*c*). The variable depth, b, is to be computed from the geometry of the water passage. It should be noted that the value of V_a/V_s, shown in Fig. 1(*c*), is of no significance in this case because of the variable depth b.

For the "free-vortex" condition, since $b = b_s + \frac{y}{d}(b_c - b_s) = \frac{b_s\, d + y\, (b_c - b_s)}{d}$,

$r = r_s - y$, and $V = \dfrac{V_s\, r_s}{r}$:

$$Q = \int_0^d V\, b\, dy = V_s\, r_s \left\{ \frac{\log_e m}{d} \left[b_s\, d + r_s\, (b_c - b_s) \right] - (b_c - b_s) \right\} \ldots (15)$$

Since $Q = V_a \dfrac{(b_c + b_s)\, d}{2}$ and $V_a = \dfrac{2\, Q}{(b_c + b_s)\, d}$,

$$\frac{V_a}{V_s} = \frac{2\, r_s}{(b_c + b_s)\, d} \left\{ \frac{\log_e m}{d} \left[b_s\, d + r_s\, (b_c - b_s) \right] - (b_c - b_s) \right\} \ldots . (16)$$

When $b_c = b_s = 1.0$, as in two-dimensional flow, $\dfrac{V_a}{V_s} = \dfrac{r_s}{d} \log_e m$ (see Eq. 10).

Circular Section.—If the cross section of the water passage is circular, with a diameter d (Fig. 2): $b = d \sin \theta$, $y = \dfrac{d}{2}$ $(1 - \cos \theta)$, and $dy = \dfrac{d}{2} \sin \theta\, d\theta$. Then

$$Q = \int_0^d b\, V\, dy = \int_0^\pi d \sin \theta\, V \frac{d}{2} \sin \theta\, d\theta$$

$$= \frac{d^2}{2} \int_0^\pi V \sin^2 \theta\, d\theta \ldots\ldots (17)$$

Let V_a = average velocity = $\dfrac{4\, Q}{\pi\, d^2}$. If the semicircle is divided into N_s segments with equal angles $\Delta\theta$,

$$V_a = \frac{4}{\pi\, d^2} \frac{d^2}{2} \sum_0^\pi V_s \frac{V}{V_s} \sin^2 \theta\, \Delta\theta$$

$$= \frac{2\, V_s}{\pi} \sum_0^\pi \frac{V}{V_s} \sin^2 \theta\, \Delta\theta \ldots . (18)$$

and

$$\frac{V_a}{V_s} = \frac{2\, \Delta\theta}{\pi} \sum_0^\pi \frac{V}{V_s} \sin^2 \theta \ldots . (19)$$

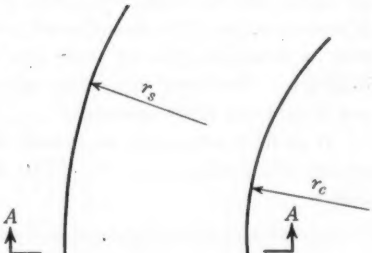

FIG. 2.—CIRCULAR WATER PASSAGE

By constructing the velocity curve (V versus y) as explained herein, values of V/V_s can be determined for the several values of the angle θ, and substituted in Eq. 19 to obtain V_a/V_s. Then V_s is determined as $\dfrac{V_a}{V_a/V_s}$; and $V_c = V_s \dfrac{V_c}{V_s}$.

For a standard bend in which $d = r_s - r_c$, if the streamlines are concentric the "free-vortex" condition may be assumed. Then, from Eq. 7,

$$V = \frac{V_s\, r_s}{r_s - y} = \frac{2\, V_s\, r_s}{(2\, r_s - d) + d \cos \theta} \ldots\ldots\ldots\ldots\ldots (20)$$

and

$$\frac{V_a}{V_s} = \frac{4\, r_s\, \Delta\theta}{\pi} \sum_{0}^{\pi} \frac{\sin^2 \theta}{(2\, r_s - d) + d \cos \theta} \quad \ldots\ldots\ldots\ldots\ldots (21)$$

Actually in a standard pipe-bend, the streamlines will follow a spiral course instead of being two-dimensional curves. Nevertheless, experiments have shown that the velocity distribution approximates that of the free vortex, although the actual value of V_c/V_s is somewhat smaller than that indicated by the foregoing theory (4*a*)(5*a*)(6*a*).

Two-Dimensional Flow with Boundaries not Fixed

As shown in the preceding section, the graphical check of the flow net can be made without difficulty when the boundaries of the water passage are fixed. If one or more of the boundaries are exposed to the atmosphere, as in discharge over a straight spillway weir, the construction of the flow net becomes more difficult. However, the same graphical check can be applied after the flow net is laid out approximately.

It is first necessary to sketch in the streamlines of the flow net freehand, as described elsewhere (1). The flow net must satisfy the following requirements:

(*a*) The radius of curvature of the streamlines must vary in conformance with the aforementioned graphical method $\left(\frac{V}{r} = \frac{dV}{dy}\right)$;

(*b*) The drop in the water surface at any section, measured below the reservoir level, must equal the velocity head of the water flowing at the surface $V^2_s/2\, g$; and

(*c*) The average velocity multiplied by the depth, measured along any potential line, must equal the rate of discharge per foot of crest length.

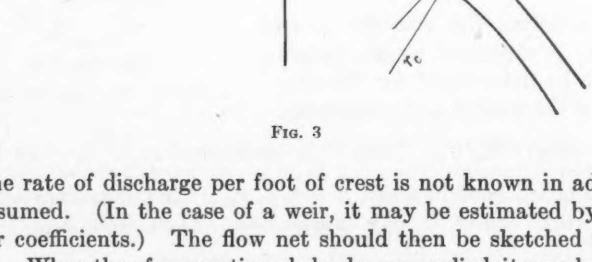

Fig. 3

Since the rate of discharge per foot of crest is not known in advance, this must be assumed. (In the case of a weir, it may be estimated by the use of known weir coefficients.) The flow net should then be sketched in freehand (see Fig. 3). When the aforementioned checks are applied, it may be impossible to make the second and third conditions agree with the first condition. This

is an indication that the assumed rate of discharge is incorrect, and a new value of discharge must be assumed. There is only one value of discharge that will permit all three conditions to be satisfied simultaneously.

In the case of a weir, the correct discharge should be determined before attempting to extend the flow net downstream from the crest. The completion of the flow net then is much simplified, as the surface velocity becomes more nearly equal to the average velocity. Where the depth of flow is relatively small, compared with the radius of curvature of the streamlines, the latter may be assumed constant; that is, $r = r_c = r_s$. Under these conditions, the theory gives $\frac{dV}{dy} = \frac{V}{r}$ in which r is constant. Then $\frac{dV}{V} = \frac{dy}{r}$; and $\log_e V = \frac{y}{r} + C$. When $y = 0$, $V = V_s$; $C = \log_e V_s$; $\log_e \frac{V}{V_s} = \frac{y}{r}$; $\frac{V}{V_s} = e^{y/r}$:

$$Q = \int_0^d V\,dy = V_s\,r\,(e^{d/r} - 1) \qquad (22)$$

The average velocity equals $V_a = Q/d$; hence,

$$\frac{V_a}{V_s} = \frac{r}{d}(e^{d/r} - 1) \qquad (23)$$

By assuming a value for d at any given section, and taking r as the radius of the concrete surface, V_a/V_s can be computed by Eq. 23. Then

$$V_s = \frac{V_a}{V_a/V_s} = \frac{Q/d}{V_a/V_s} \qquad (24)$$

This value of V_s can be checked by the condition that $V^2{}_s/2\,g$ must agree with the elevation of the water surface at the given section. Having determined V_s, the next step is to compute V_c: Since $V = V_s\,e^{y/r}$,

$$V_c = V_s\,e^{d/r} \qquad (25)$$

Check with Experimental Weir Nappes.—The writer has checked the methods described herein for testing a flow net by applying them to the case of a sharp-crested weir with aerated lower nappe. Curves for the upper and lower surface of the freely-discharging jet can be plotted by means of the table prepared by E. W. Lane, M. Am. Soc. C. E. (7). At any section through the jet, the elevations of the upper and lower water surfaces may be determined with respect to the reservoir surface (H_s and H_c in Fig. 3). Since atmospheric pressure prevails on both surfaces, $V_c/V_s = \sqrt{H_c/H_s}$.

By measuring the thickness of the jet and the radius of curvature of the upper and lower nappes at the given section, the value of V_c/V_s may also be computed by the graphical or analytical methods described herein. The results, as obtained by the writer, give a very close check with the values of V_c/V_s obtained from the velocity heads.

Three-Dimensional Flow with Boundaries not Fixed

As in the case for fixed boundaries, the methods described herein for two-dimensional flow can be extended to three-dimensional flow with one or more

boundaries exposed to the atmosphere. An example of this condition is the "morning-glory" type of spillway, or a sharp-crested weir which is circular in plan (8).

The flow net in this case is to be constructed in a manner similar to that described for two-dimensional flow, except that, in determining the actual velocities, the shape of the water passage must be taken into account, as described herein for fixed boundaries.

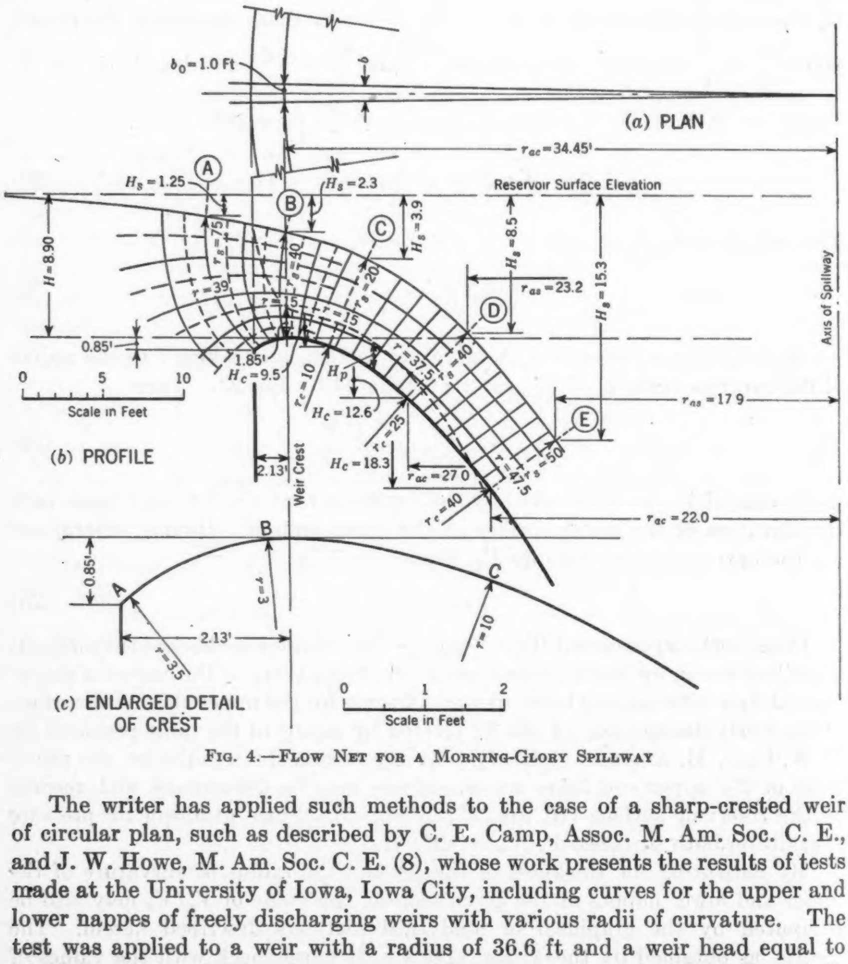

FIG. 4.—FLOW NET FOR A MORNING-GLORY SPILLWAY

The writer has applied such methods to the case of a sharp-crested weir of circular plan, such as described by C. E. Camp, Assoc. M. Am. Soc. C. E., and J. W. Howe, M. Am. Soc. C. E. (8), whose work presents the results of tests made at the University of Iowa, Iowa City, including curves for the upper and lower nappes of freely discharging weirs with various radii of curvature. The test was applied to a weir with a radius of 36.6 ft and a weir head equal to 10.45 ft. After plotting the upper and lower nappe curves, the values of H_c and H_s were scaled for several sections through the jet and V_c/V_s was computed ($= \sqrt{H_c/H_s}$). The graphical and analytical methods previously described were then used to compute values of V_c/V_s for the same sections, using values of the radii of curvature and thickness of the jet scaled from the plotted

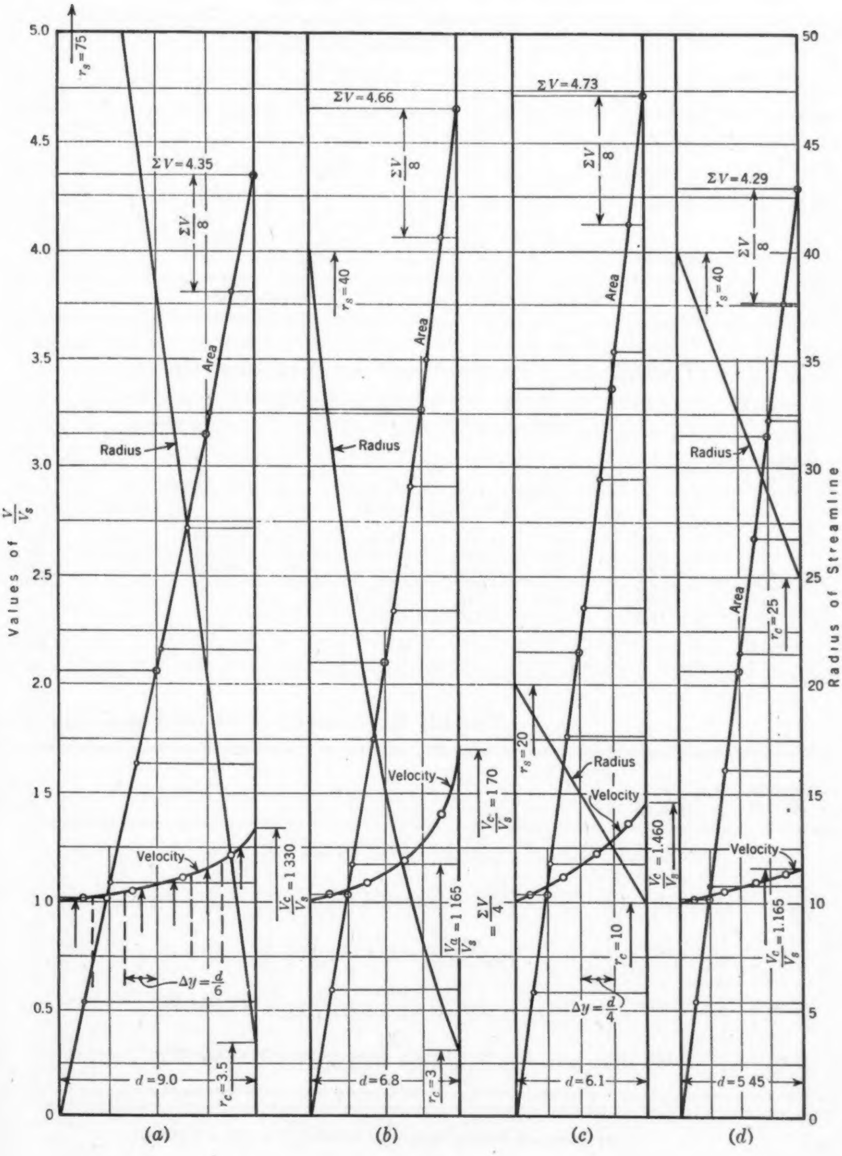
Values of V/V_s
Radius of Streamline
ΣV=4.35
ΣV/8
Area
Radius
Velocity
Δy=d/6
d=9.0
r_s=75
r_c=3.5
V_c/V_s=1.330
ΣV=4.66
r_s=40
V_c/V_s=1.70
V_a/V_s=1.165
d=6.8
r_c=3
ΣV=4.73
r_s=20
V_c/V_s=1.460
=ΣV/4
r_c=10
Δy=d/4
d=6.1
ΣV=4.29
r_c=25
V_c/V_s=1.165
d=5.45
(a)
(b)
(c)
(d)

FIG. 5

TABLE 1.—Illustrative Example; Basic Data

(Flow Q = 99 Cu Ft Per Sec)

Symbol	Sections (See Fig. 4)				
	A	B	C	D	E
r_s	75	40	20	40	50[a]
r_c	3.5	3	10	25	45[a]
d	9.0	6.8	6.1	5.45	5.1
Δy	1.50	....	1.53	....	....
r_{ac}	....	....	....	27.0	22.0
r_{as}	....	....	....	23.2	17.9
d/r	....	....	....	0.1455	0.1073
b_c	....[b]	1.0	....[b]	0.784	0.639
b_s				0.674	0.520

[a] Average $r = \frac{1}{2}(r_s + r_c) = 47.5$. [b] See Table 2.

TABLE 2. —Computations for Variable Depth

Segment No. (Fig. 4)	Section A				Section C			
	r_a	$b = \frac{r_a}{34.45}$	$\frac{V}{V_s}$ Fig. 5(*a*)	$b \times \frac{V}{V_s}$ (Eq. 12)	r_a	$b = \frac{r_a}{34.45}$	$\frac{V}{V_s}$ Fig. 5(*c*)	$b \times \frac{V}{V_s}$ (Eq. 12)
1	39.5	1.145	1.005	1.150	29.8	0.865	1.03	0.890
2	39.4	1.142	1.035	1.182	30.4	0.882	1.12	0.988
3	39.0	1.130	1.055	1.192	31.0	0.900	1.22	1.092
4	38.5	1.120	1.085	1.215	31.7	0.920	1.37	1.260
5	37.9	1.100	1.140	1.252	....	....	....	....
6	37.0	1.072	1.230	1.320	....	....	....	....
Total	....	....	....	7.311	....	....	....	4.230

TABLE 3.—Computation of Flow Net Values

Section (Fig. 4)	$\frac{V_c}{V_s}$ (Fig. 5)	V_s	$\frac{V_a}{V_s}$ (Fig. 5)
A	1.33	$\frac{99}{1.5 \times 7.31} = 9.02$ (Eq. 12)	....
B	1.70	$\frac{14.58}{1.165} = 12.52$[b] (Eq. 5*a*)	1.165
C	1.460	$\frac{99}{1.53 \times 4.23} = 15.4$ (Eq. 12)	....
D	1.165[a]	$\frac{2 \times 99}{(1.165 \times 0.784 + 0.674)\, 5.45} = 22.9$ (Eq. 27)	....
E	1.113	$\frac{2 \times 99}{(1.113 \times 0.639 + 0.520)\, 5.1} = 31.6$ (Eq. 27)	....

[a] Approximate computation: $\frac{d}{r} = 0.1455$; $\frac{V_c}{V_s} = e^{d/r} = 1.156$.

curves. As with the straight weir, the agreement with the velocity head computations was very close, particularly in the region of the jet where the streamlines have large radii of curvature. Near the weir crest the agreement

was less satisfactory, but probably within the limits of accuracy of the curves used by Messrs. Camp and Howe (8) which are drawn at a rather small scale.

Example

To illustrate the methods outlined herein, the case of a morning-glory spillway weir will be given. The profile of the crest is shown in Fig. 4, the high point of the weir being located in plan on a circle whose radius r_{ac} is 34.45 ft, and whose circumferential length is 217 ft. The discharge capacity is assumed as 21,500 cu ft per sec, or 99 cu ft per sec per foot of crest. The analysis is made for a sector 1.0 ft wide at the crest.

Because of the circular plan of the spillway, a section through the water jet, such as section D in Fig. 4, will be essentially trapezoidal in shape, the depth of the section at any point (normal to the paper) being equal to $r_a/34.45$, in which r_a is the radius measured from the axis of the spillway.

Preliminary studies indicate that the head over the spillway crest required to discharge 99 cu ft per sec per ft will be about 8.90 ft. Using this head, the flow net is sketched in as shown in Fig. 4. The radii of curvature of the water surface, the mid-streamline, and the concrete surface are scaled at various sections. For this purpose, a template on tracing paper showing curves of various radii is convenient.

The graphical check for section A, Fig. 4, is shown in Fig. 5(*a*), in which the area curve is divided into eight stream channels ($N = 8$). The computations for V_s and V_c are given in Tables 1, 2, and 3, the width d being divided into six equal segments ($N_s = 6$). As a check on the accuracy of the flow net, it will be noted that $V^2_s/2\,g$ should equal H_s. The pressure head at the con-

(Q = 99 Cu Ft Per Sec)

$\frac{V^2_s}{2\,g}$	V_c (Eq. 5*b*)	$\frac{V^2_c}{2\,g}$	H_s (ft) (Fig. 4)	H_c (in feet) (Fig. 4)	H_p (in feet) $H_c - V^2_c/2\,g$	Section (Fig. 4)
1.26	9.02 × 1.33 = 12.0	2.24	1.25	9.75	9.75 − 2.24 = +7.51	A
2.4	12.52 × 1.7 = 21.3	7.05	2.3	8.90	8.90 − 7.05 = +1.85	B
3.7	15.4 × 1.460 = 22.5	7.9	3.9	9.5	9.5 − 7.9 = +1.6	C
8.2	22.9 × 1.165 = 26.7	11.1	8.5	12.6	12.6 − 11.1 = +1.5	D
15.5	31.6 × 1.113 = 35.2	19.2	15.3	18.3	18.3 − 19.2 = −0 9	E

[b] $V_a = \frac{99}{6.8} = 14.58.$

crete surface, H_p, is computed as equal to $H_c - \frac{V^2{}_c}{2\,g}$, and is plotted in Fig. 4 vertically from the concrete surface.

Section B is approximately vertical, and hence rectangular, with constant depth, $b = 1.0$. The graphical check is given in Fig. 5(*b*); and the computations are shown in Tables 1 to 3. In this case, V_a/V_s and V_c/V_s are obtained from the graphical check.

The computations for section C are made in the same manner as for section A, with d divided into four equal segments. The graphical check is given in Fig. 5(*c*).

At section D the graphical check is given in Fig. 5(*d*). Here the velocity curve is practically a straight line and the cross section of the stream passage is a trapezoid. The discharge may then be computed as

$$Q = (V_c\, b_c + V_s\, b_s)\,\frac{d}{2} \qquad (26)$$

or

$$V_s = \frac{2\,Q}{\left(\frac{V_c}{V_s}\, b_c + b_s\right) d} \qquad (27)$$

V_c/V_s being read from Fig. 5(*d*). The approximate method, using a constant radius of curvature for the streamlines, may also be used, in which case $V_c/V_s = e^{d/r}$. As shown in the computations, this gives an error of less than 1% in the value of V_c/V_s, even though there is an appreciable difference between r_c and r_s.

At section E the streamlines have practically a constant radius, and the approximate method is used to compute V_c/V_s. Then V_s is computed by Eq. 27 as in section D; thus:

$$\begin{aligned} \log_{10} e &= 0.43429 \\ & \ \times\ \underline{0.1073} \qquad \left(=\frac{d}{r};\ \text{see Table 1}\right) \\ \log e^{d/r} &= 0.04660 \\ e^{d/r} &= 1.113 = \frac{V_c}{V_s} \qquad \text{(see Eq. 25)} \end{aligned}$$

The graphical checks for sections A to D, inclusive, give the theoretical spacings of the streamlines, which are used to check the construction of the flow net. If there is any appreciable change to be made in the curvature of the streamlines, the graphical check should be revised accordingly. In the preliminary construction of the flow net it may be found that the computed $V^2_s/2\,g$ at any section does not agree with H_s shown on the flow net. The position of the water surface must then be shifted until these values agree.

In this example, considerable time is required to determine the correct head (H) over the spillway crest, for the given value of discharge. Several trial arrangements of the flow net between section A and section C must be laid out, using different values of H, until the values of H_s and $V^2_s/2\,g$ are brought into agreement. The flow net is not extended beyond section C until the correct value of Q has been determined.

This example was selected intentionally to illustrate the complications of a three-dimensional problem with water surface exposed to the atmosphere. A two-dimensional problem, such as a weir with straight crest, or a problem

involving fixed boundaries for the water passage, would be appreciably easier to construct.

Summary

The writer has found by experience that the methods outlined herein greatly facilitate the construction of a flow net, and permit much more precise velocity determinations than can be obtained merely by freehand sketching of the streamlines. The experience of other investigators (1), and the check computations, made by the writer, on the results of experiments on straight and curved sharp-crested weirs, lead to the conclusion that, when a flow net is carefully constructed and checked by the methods presented herein, the resulting computed velocities and pressures will be a reasonable indication of those to be expected in the prototype structure. However, it should be recognized that the flow-net method is not applicable in cases where friction and impact or eddy losses may be expected to have a controlling effect on velocities and pressures.

APPENDIX I

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(7) **"Handbook of Applied Hydraulics,"** by Calvin Davis, Editor, McGraw-Hill Book Co., Inc., New York and London, 1942, p. 338.

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APPENDIX II

Notation

The following letter symbols, used in the paper, conform essentially to American Standard Letter Symbols for Hydraulics,[3] prepared by a Committee

[3] ASA—Z10.2—1942.

of the American Standards Association, with Society representation, and approved by the Association in 1942:

b = breadth of flow (Fig. 1(b)), b_s and b_c, respectively, being breadths at the outside and inside of the curved stream channel;
C = a constant (see Eq. 6);
d = depth, or total maximum width of a water passage measured along a curved potential line (see Fig. 1);
e = base of Napierian logarithms (see Eq. 8);
g = acceleration of gravity;
H = elevation of reservoir surface above weir crest (see Fig. 4);
H_c and H_s = respectively, the elevation of the lower and upper surfaces of the water at any section of a jet, measured downward from the reservoir surface:
H_p = pressure head on masonry surface (see Fig. 4);
m = the ratio r_s/r_c (Eq. 10);
N = total number of hypothetical channels in a stream (Fig. 1(a));
N_s = number of hypothetical segments for velocity curve, Fig. 1(c);
Q = total discharge, or total area under the curve A_2C_2, Fig. 1(c);
q = discharge in any given stream channel, or Q/N (Eq. 2c);
r = radius of curvature of a stream, the radii for individual channels (Fig. 1(a)) being designated by appropriate subscripts; the radii of the boundaries of the jet are denoted by r_s and r_c (Fig. 1(a));
r_a = radius of "morning-glory" spillway measured from the axis of the spillway shaft to a point in any section of a jet, the radii to the lower and upper surfaces at any section being designated by r_{ac} and r_{as}, respectively (see Fig. 4);
V = velocity of flow at any point P, Fig. 1(a), in the water passage, the velocity for individual channels (Fig. 1(c)) being designated by appropriate subscripts; average velocity is designated V_a, and the velocity at the inner and outer surfaces of any section is designated V_c and V_s, respectively:
V_1 = average velocity in stream channel 1 (see Fig. 1(c));
$(V_1)'$ = value of V at the midpoint of segment 1 of the velocity curve;
y = a part of d measured from the outer curved boundary toward the center of curvature (see Fig. 1(a));
y_0 = width of any single segment of area curve, Fig. 1(c)), equals d/N_s;
Δy_1 = width of stream channel 1 (Eq. 13);
θ = angle measured at the center of a circular channel (Fig. 2):
$\Delta\theta$ = central angle of the arc subtended by a segment of the velocity curve (Fig. 1(c)).

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
REPORTS

DESIGN AND CONSTRUCTION OF AIRPORT RUNWAYS AND FLIGHT STRIPS

PROGRESS REPORT OF A COMMITTEE OF THE HIGHWAY DIVISION

Introduction

A discussion of the problems and factors that the Committee is considering, with some indication of the conception of the scope of the project, is contained in this progress report. Recognizing the significance of its title, the Committee has attempted to select those matters for study which, in its judgment, would be of interest to the greatest number of engineers.

As is the case with nearly everything that the engineer is called upon to build, the design of an airport or "flight strip" develops into a compromise between the desire to construct a facility that will be bigger and better than any heretofore contemplated and the amount of funds that can be justified and obtained for the project. If soundly planned, the compromise takes the form of deferring, until a later date, certain improvements contemplated in the master plan, but, at the same time, provides sufficient flexibility to make possible the extension of the runways, as well as other facilities, should it become necessary, without entailing unreasonable expense in acquiring additional land for the airport and control over such approach zones as may be necessary to assure safe operation. Failure to provide the flexibility necessary to permit the extension of the runways and the other facilities may result in newly constructed airports approaching the beginning of obsolescence almost upon their completion.

In view of the fact that the design of the runway and runway pattern, in the original construction as well as the possible future extension, is restricted by the limitations of the site, consideration must be given to the runway when the selection of the site is first undertaken. Many cities now find themselves with commercial airports which are too restricted in size for adequate air transport operation and cannot be expanded, with the result that, in order to have an adequate airport, a new site must be selected and a new airport constructed. In most cases the restriction is due to the inability to extend the runways to accommodate even present needs. In other cases, the original layout of the

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by October 1, 1944. Progress Reports are published in *Proceedings* only.

runways prohibits the construction of parallel runways, thereby restricting the capacity of the airport.

Recognizing that the runway is one of the determining factors in the selection of a site for an airport or flight strip, runway design should receive attention when the selection of the site is first contemplated. All the skill of the most competent engineers cannot extend a runway to meet increased traffic demands if an error was made in selecting an inadequate site when the airport was constructed.

Runway Patterns

Among the factors that require study are: The number of runways and taxiways; their arrangement with regard to each other and their arrangement with regard to the terminal building, hangars, and other unavoidable obstacles off as well as on the site; the ease and speed of passenger access to and from planes; and the time required for planes to travel to and from the runways, the terminal building, and between the terminal building and the hangars. Although each airport presents an individual problem, certain fundamental principles apply to all.

A layout of runways that will provide for rapid landing and take-off of airplanes and prevent loss of time in the air, as well as on the ground, thereby affording the airport a maximum plane capacity, is a matter of great concern to operators of airports and the operators of commercial air lines. To the former, it may mean one less headache; to the air-line operator, it is a matter of dollars and the maintenance of better schedules. The length of the runways, their width and number, suitable patterns for runway layout, and many other similar questions are being studied by the Committee.

Runway Requirements

The development of the airplane and the airport go "hand in hand." One is of necessity the complement of the other. Much attention has been directed recently to the helicopter and its effect on landing field design, particularly with regard to the smaller fields for private use. As recently as 1929 or 1930 it was anticipated that improvements in the conventional type of airplane would enable planes to land and take off at slower speeds and thus require shorter runways. Time has demonstrated, for obvious reasons, that longer and longer runways have been required.

On the other hand, no doubt, the question as to whether runways are being designed which are longer than future requirements would justify is of great concern to many engineers. They are not only conscious of the possibilities of the helicopter, but are aware of the development of take-off assisting devices (the various forms of the catapult and jet propulsion), as well as devices to make possible faster deceleration (the reversible propeller, improved brakes in combination with the tricycle landing gear, improved flaps, etc.). Such devices, of course, would have the tendency to reduce the required length of the runways.

If the answer to the question as to whether longer or shorter runways will be required were known now, airports for the future could be designed with greater assurance.

With longer runways, planes can be built to operate at lower cost. On the other hand, to provide ground facilities to accommodate such planes would cost considerable money. These two factors of cost must be reconciled—the cost of airplane operation versus the cost of airport construction and maintenance.

Runway lengths and patterns of future airports should be designed with an understanding of their relationship to probable future aircraft operating requirements.

Due to the performance characteristics of the airplanes and the requirements enforced to assure safety in the event of engine failure during take-off, the cost of providing the land required for site, paving, and lighting these long runways, as well as the cost of their maintenance, continues to grow. It should be remembered, however, that the load-carrying characteristics and performance are sacrificed if a plane must be designed for use on runways that are not sufficiently long. The answer may be found through regulation.

Weight of Planes

What will be the weight of the heavy transport airplane in the postwar period? To what extent will the heavier type be used and which airports will receive scheduled stops by the heavier type of airplane? Will this type, requiring the heaviest and longest runways, be restricted, for economic reasons, to the airports of the large cities?

Landing Gear

Will the heavy plane of the future be equipped with single or multiple rubber-tired wheels and, if so, how will they be spaced; or, will some form of track type landing gear be used? The difference in load distribution, depending upon the type of landing gear, will greatly affect the design. As long as the answer to this question remains unknown, one of the important design factors will remain a variable.

Wing Loadings

With the increase in the wing loading factor in aircraft design, thus requiring longer take-offs and landing runs, it has been necessary to expand airports in all directions.

Runway Design

In the past year or so, considerable work has been done to put airport design on a "going" basis. This is particularly true of runway pavements. However, as the period is so short during which intensive thought has been given to the subject, and, as sufficient time has not elapsed to evaluate the behavior of runways under heavy loadings, published information on the subject is meager. No doubt, much valuable information has been developed which has not yet found its way into print. This is recognized by the Committee and a special effort will be made to obtain such information.

There are many problems confronting the engineer called upon to design a runway other than that of determining the thickness of the pavement. However, no phase of the problem is at present the subject of more discussion and

controversy. This has been accentuated by the rapid development of heavier and heavier planes, particularly for military purposes. The dearth of available data from which conclusions may be drawn is a handicap in the work of this Committee. The extensive research work in laboratory and field now being done should prove of great value to the Committee, and it is hoped that its final report will contain information which will prove helpful in this phase of the problem.

With minor exceptions, types of pavement suitable for airport runways are the same types of pavement that have been found suitable for modern highways and, therefore, the experience gained in highway construction is invaluable in connection with this phase of the problem. However, the wheel loads of the heavy airplanes have introduced a factor for which there is no previous experience. The intensive use by heavy military planes, given many runways of various types and designs, has provided an accelerated service test from which, no doubt, excellent data will be obtained.

By means of field and laboratory investigations now being conducted, the correlation of results obtained by the use of various tests on soil and pavements, and the correlation of these tests with the various design theories, fundamental data should be obtained on the load-supporting value of nonrigid pavements of various thicknesses, with respect to various base-course thicknesses and degree of subgrade support. Likewise, investigations relating to the rigid type of pavement should produce information of value in checking theories of design.

Small Landing Fields

According to Charles I. Stanton, Administrator of Civil Aeronautics, about 3,000 new fields will be needed in the postwar period, and, although a number of large fields are required, the greater part will be small ones. This will call for large sums of money, not only for construction, but for maintenance, year after year. Although the design of a small field has little appeal to the engineer as compared to the spectacular job of building a large airport costing many millions, the cost for the small fields will aggregate large sums and their design is worthy of the best thought that can be applied to them.

The highway system in the United States represents a case in point. In the early days of highway construction, the highway engineer was so engrossed in building only the higher types that he gave no thought to what is now known as the "low-cost types." Every one is aware of the results that have been obtained since the development of low-cost roads was begun. Just as the percentage of express highways is small, and will remain small, compared with the total mileage of all highways, so will the number of small fields form a very large part of the entire airport system.

Robert H. Hinckley, former Chairman of the Civil Aeronautics Authority and later Assistant Secretary of Commerce for Air, has said in a recent statement concerning the need for adequate landing areas for private flyers:

> "An airplane isn't truly useful if one must, for the lack of an airport located at the place where one wants to go, takeoff and fly around his takeoff point just for the sport of flying.

"Just as the automobile was not truly useful until highways were built to places people wanted to go, neither will the airplane be able to take its rightful place in the everyday lives of our people until adequate airports and navigational aids are available to those who want to fly their own planes on business, pleasure or shopping trips.

"True, we will have a great many more airports after the war than we ever had before. True, we have an excellent national airways system. And they [airways] belong to all the people—not just the airlines, the Army, the Navy and the Department of Commerce. If, for safety reasons, it is not feasible for the private flyer to use these facilities, then all the people must provide whatever else is necessary to make private flying useful, safe and easy. There should be landing strips placed alongside the nation's highways in such locations that they will be convenient stopping places for these flyers who will fly over, instead of riding on, the highways—using them only for navigation purposes. Markers visible from the air should be placed along these highways."

A form of landing strip, which is known as the "flight strip," has been constructed for military purposes adjacent to public highways. In the Federal Highway Act of 1940, Congress authorized the Commissioner of Public Roads, in cooperation with the State Highway Departments of the respective states, upon the request of any state, to investigate the location and development of flight strips adjacent to public highways or roadside development areas, for the landing and take-off of aircraft. This was followed in 1941 by an appropriation in the amount of $10,000,000 for the construction of flight strips in cooperation with the Army Air Forces. These strips were built where it was necessary to have a landing field with a long runway and clear approaches with a flat glide angle, but where the construction of a regular airport was not justified.

To date (1944), twenty-six flight strips have been built in the United States and eight in Canada along the Alaska Highway. Those in Canada were constructed by the United States Engineer Department, after the Public Roads Administration had selected the sites, made the surveys, and completed most of the plans.

The Committee will take full advantage of the opportunity presented by this type of landing facility to inform itself of the extent to which it may be utilized.

It appears to be the consensus that all planes, including the light, private airplanes, will soon have the tricycle landing gear, which will make them safer for landing. Planes thus equipped will also effect a saving in the cost of the construction of airports. The ease with which a plane equipped with the tricycle landing gear can be handled in a cross wind, as compared to that with the conventional type of landing gear, is well known.

From information thus far obtained, the percentage of time that a single runway can be used, without requiring the plane to land in a cross wind of more than 20 miles per hr, appears to be quite high. The Committee is collecting such information.

The Integration of Airport and Highway Planning

After the war, aviation faces the greatest development in the history of transportation. To assure its successful and rapid development, ground

facilities must keep pace with the improvements in the airplane. Also, it seems inevitable that the system of airports of a few years hence will be integrated closely with the highway system. The extensive planning now being done by the Public Roads Administration, under the leadership of Thomas H. MacDonald, Hon. M. Am. Soc. C. E., in cooperation with the State Highway Departments, affords a timely opportunity for joint planning of both forms of transportation.

William A. M. Burden, Special Aviation Assistant to the Secretary of Commerce, recently stated:

> "Comprehensive and farsighted planning, stressing proper integration of air and surface transport routes, and teamwork among federal, state and local agencies, are the common denominators for success in the planning of airports. * * * Insofar as planning for future airports is concerned, * * * appropriate surface access highways must be considered. In many cases, the choice must be made between expending relatively large sums of money on express highways to reach cheap outlying sites, or concentrating the expenditure on higher priced real estate for the airport closer to the center of town and saving on the highway investment."

Summary

An attempt has been made in this report to describe what the Committee is considering and studying. However, in this fast moving field of aviation, changes can and do take place overnight.

Respectfully submitted,

Jay Downer George M. Shepard
Roy W. Crum William B. Catchings
Frederick E. Schnepfe, *Chairman*

Committee of the Highway Division on Design and Construction of Airport Runways and Flight Strips

January 21, 1944.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

THE HYDRAULIC JUMP IN SLOPING CHANNELS

Discussion

BY CARL E. KINDSVATER

CARL E. KINDSVATER,[31] JUN. AM. SOC. C. E.[31a]—Observers apparently do not agree on a definition for the true hydraulic jump. Mr. Stevens discards the jump on a slope, because it is "a mere plunging of water into a pool," and because,

> "As the slope steepens, the jump loses its identity entirely and becomes something else about which little is known."

He quotes Messrs. Bakhmeteff and Matzke[3] to support his opinion:

> "The live jet 'plunges' into the tailwater and within the steep section follows the slope downward with comparatively slow expansion and obviously relatively small losses."

A typical opinion, in which the jump below steep slopes is qualified as a "drowned-out" jump, was expressed by Professor Posey,

> "A drowned-out jump cannot be classed as a true jump since it has lost its ability to lower velocities safely within a short distance along the stream."

Thus, the criterion for a true jump would be its ability to destroy energy, and the reason for disqualifying the hydraulic jump in sloping channels is an opinion concerning its suitability as a stilling basin device. The disqualification is not based on proof that a similar manner of analysis will not apply to all jumps, free or "drowned out," in sloping or horizontal channels.

It is hazardous to attempt a general definition of a phenomenon having so many variations of form as the hydraulic jump in open channels of any slope. Fundamentally, the hydraulic jump is marked by an abrupt increase in the depth of flow. It is created when a stream of water flowing at less than critical depth is retarded by a stream of water flowing at a depth greater than

NOTE.—This paper by Carl E. Kindsvater was published in November, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1943, by Joe W. Johnson, and Karl R. Kennison; May, 1943, by J. C. Stevens, and C. J. Posey; June, 1943, by Jerome Fee, and September, 1943, by Frank S. Bailey, and G. H. Hickox.

[31] Associate Hydr. Engr., Hydroelectric Section, U. S. Engr. Office, Little Rock, Ark.

[31a] Received by the Secretary March 13, 1944.

[3] "The Hydraulic Jump in Sloped Channels," by B. A. Bakhmeteff and A. E. Matzke, *Transactions*, A. S. M. E., Vol. 60, 1938, Paper HYD-60-1, pp. 111–118.

critical. To form a stable jump in horizontal channels, the depth of flow at the downstream section must be exactly equal to the upper conjugate depth. When a jump is formed on a slope, the depth of water at the end of the slope may exceed the upper conjugate depth, but the stream will continue to expand beyond the end of the jump because of the increasing depth and the resulting transformation of energy. For any given conditions of initial depth and discharge, there is only one corresponding conjugate depth, and any change of slope beyond the point where this depth occurs will have negligible effect on the location of the beginning of the jump, its length, or its form. Thus, the selection of economic stilling basin dimensions might depend on the upper conjugate depth. This paper did not intend to propose the hydraulic jump in sloping channels as a stilling basin device. It has been used for that purpose, however, and apparently with some success. Therefore, regardless of a difference in opinion concerning the name of the phenomenon, computation of its essential dimensions is frequently desirable. The analysis proposed by the writer appears to serve this purpose. The length of the roller, as defined by the writer, is a fair approximation of the length from the toe of the jump to the section of upper conjugate depth. In addition, the agreement between measured and computed depths for jumps occurring on a 1-on-6 slope is adequate proof of the practical value of the analysis.

The method described for determining the end of the roller might possibly yield unreliable results for very steep slopes. Mr. Hickox' discussion indicates, however, that this limit must be beyond a slope of 1 on 3. The writer's treatment of Case 2 jumps on 1-on-6 slopes may prove invalid for other slopes. Obviously, other limitations might be imposed by the form of the structure on which the jump must occur. For those cases where the path of the water under the jump is materially influenced by the shape of the structure, or where it is obstructed by impact devices (baffle piers, sills, etc.), the proposed analysis of the simple phenomena certainly is not a substitute for model studies.

In the foregoing quotations, the hydraulic jump which forms on slopes greater than about 1 on 10 is described as an occurrence in which the live jet plunges under the roller in a manner distinct from the behavior of the jump in horizontal channels. Under the heading, "The Hydraulic Jump in Sloping Channels: Limitations to Analysis," the writer expressed the opinion that the jump on a slope is generally similar to the jump in horizontal channels. In support of this opinion he submitted two large photographs which lost some of their effectiveness in reproduction (see Figs. 3 and 4). In another attempt for this closing discussion, he devised a miniature laboratory in his family bathroom and produced the four views in Fig. 21. The test flume was approximately 2 in. wide and 24 in. long. The photographs were taken with an exposure of 1/100 sec. The kinetic flow factor for all photographs was estimated to be approximately 35. In these short exposures, action is revealed which is not apparent to the naked eye. "White water" becomes a pattern of air bubbles and water. Slowly moving bubbles are caught as if motionless; rapidly moving bubbles make a visible streak, the length of which is a measure of local velocity components. Because the flume was so unusually narrow, the

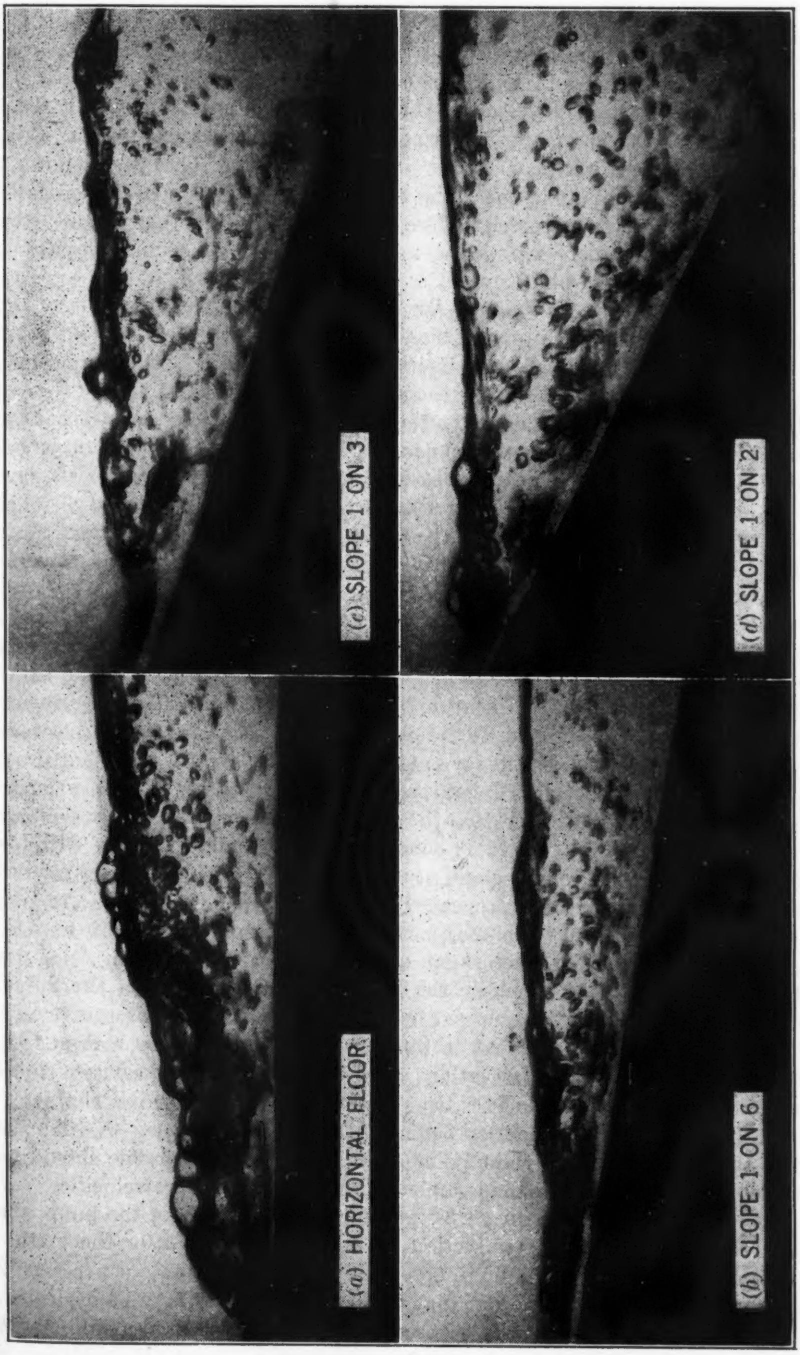

FIG. 21.—VIEWS OF THE HYDRAULIC JUMP

number of air bubbles involved in the photographs is comparatively small. The pictures illustrate instantaneous typical sections rather than average conditions. The four views in Fig. 21 show clearly a general confusion of path lines and a wide range of velocities throughout the jump. Fig. 21(*c*), for a 1-on-3 slope, shows a remarkable dispersion of high-velocity currents. Fig. 21(*a*), for the horizontal floor, is not a good photograph because it shows a temporary splash of bubbles near the toe. The distribution of velocities in Fig. 21(*a*) does not appear widely different from that for the other views. Any evidence of the distinctive plunging action described by some observers is difficult to find in these photographs.

Length of the Hydraulic Jump.—Argument concerning the definition of the downstream limit of the jump is made confusing by the tendency of some writers to think of the jump only in its relation to stilling basin design. As defined by the writer, the length of the jump is the horizontal length of the top roller. This definition was adopted after careful investigations showed that the end of the roller was the only section near the end of the rapid expansion which could be easily defined and readily observed, giving consistent results when recorded by different observers. For Case 3 jumps, the observed end of roller usually occurs near the point where the water surface first reaches its maximum height. Actually, this "point" is a fluctuating "zone," and argument concerning its exact definition is superfluous.

An important assumption in the writer's analysis, as well as in the classic analysis of Case 1, is that the velocity distribution at section 2 is reasonably uniform, in order that (*a*) the average velocity, $V_2 = Q/A_2$, will yield a satisfactory evaluation of the momentum in that section, and (*b*) the pressure distribution may be assumed to be hydrostatic. The agreement between computed and measured depths for both Case 3 and Case 4 is sufficient proof that the assumption is valid for practical use. As a matter of fact, velocity measurements at section 2 show that the distribution of velocities is not uniform at the end of the roller. Sporadic concentrations of higher-velocity currents prevail considerably beyond the end of the roller. Throughout the long series of tests described by the writer, repeated attempts were made to define a section where destructive bottom velocities ceased. The paths of entrained air bubbles were carefully observed to determine a section where the bottom velocities were no longer sufficient to carry the bubbles along the floor of the flume. Several hundred observations served only to prove that two observers could not agree on the location of that section, usually disagreeing by several feet. Attempts to analyze the observations were fruitless. The presence of local concentrations of erosive velocities beyond the end of the roller is a characteristic of the jump on a level floor as well as of the jump on a slope. Neither the end of the roller nor the highest point on the water-surface profile should be accepted as a zone marking the complete dissipation of erosive velocities.

There is no law of mechanics which requires that the end of the jump, the end of the roller, or any other section be taken as section 2 for the mathematical analysis of the hydraulic jump. The length of the jump is necessarily made as short as possible in order that the boundary losses may be neglected. This does not signify that the pressure-momentum equality could not be

verified at any section between the beginning and end of the jump. Quite to the contrary, Kazimierz Woycicki,[32] working with the discharge of water below rectangular sluice gates, has shown that for both the free and the "covered" jump, the fundamental pressure-momentum equality can be verified for any intermediate section, as long as the direction and magnitude of the component velocities and pressures are properly considered. Any successful mathematical solution for the hydraulic jump in sloping channels hinges not only upon the analytical evaluation of forces and velocities at the end sections but also upon the experimental evaluation of forces on the sloping floor under the body of the jump.

Mr. Stevens' comments concerning the length of the jump appear to be somewhat contradictory. For example, he states that "The length of the jump is quite arbitrary except that it should not be assumed too short." Then, recognizing that the length varies with the kinetic flow factor, he adopts for his general analysis the involved formula proposed by Professor Ivanchenko for jumps on a level floor. If the quotation cited is correct, would it not be better to assume that the length ratio might be taken as a "safe" constant? Such was the assumption made by Messrs. Bakhmeteff and Matzke,[3] as well as by Mr. Kennison. However, as demonstrated by the experimental data—both Mr. Yarnell's and Mr. Hickox'—the length of the jump on a slope is dependent on the degree of slope as well as on the kinetic flow factor. Curves of L_R/d_2 derived from the experiments are quite well defined for a practical range of λ-values, making unnecessary the use of an arbitrary "safe" length, which to be safe must be excessive. Regardless of the definition for section 2, the slope of the apron is obviously an important factor governing the length of the principal expansion within the jump.

Fig. 16, based on an algebraic extension of Mr. Stevens' generalized analysis, does not show a true variation of length as a function of slope, since Eq. 43, taken from Eq. 29, assumes the length of the jump to be primarily a function of the kinetic flow factor.

Classifications of the Hydraulic Jump.—Mr. Kennison questions the title of the paper on the grounds that the "fifth case," in which the water surface downstream from the jump is parallel to the slope of the floor, has been omitted. Although the limits imposed on the length of descriptive titles often lead to ambiguous expressions, the writer believes that the scope of the paper is adequately described by the title. Cases 3 and 4 are concerned with jumps entirely on the sloping floor. Case 1 is necessarily included as a basic case. Case 2 is a practical intermediate classification. The fifth case, as defined by Mr. Kennison, occurs only infrequently and only on mild slopes, where d_2 is ordinarily the normal depth of flow. For practical purposes, any analysis which leads to a satisfactory solution of Case 4 will also yield a satisfactory solution for the fifth case. In the summation of pressure and momentum at section 2, the effect of substituting $V_2 \cos \alpha$ for V_2 would, because of the relatively small magnitude of $\frac{Q\,\gamma\,V_2}{g}$ compared to P_2, be negligible. Thus, the value of

[32] "The Hydraulic Jump and Its Top Roll and the Discharge of Sluice Gates," by Kazimierz Woycicki, translation by I. B. Hosig, *Technical Memorandum No. 435*, U. S. Bureau of Reclamation, Denver, Colo., Chapters III and IV.

d_2 computed from Eq. 14 may be expected to agree satisfactorily with measured vertical depths at the end of a jump in a continuously sloping channel. All of the cases defined by the writer are basic to the extent that they include the four principal forms of the jump within a full range of tailwater levels. As stated by Professor Posey, and, as defined by the writer, Case 3 is really the primary case of the hydraulic jump in sloping channels. The agreement between experimental coefficients for Case 3 and Case 4 demonstrates that the distribution of velocities in section 2 can be neglected, and further obviates the fifth case suggested by Mr. Kennison.

Verification of the Analysis by Application to Other Slopes.—Mr. Johnson suggests that all of the writer's four basic cases are contained in Colonel Rindlaub's data in Fig. 10. Not only is this true, but Case 3, in which $l = \frac{L_R}{\cos \alpha}$, can be defined in Fig. 10 with remarkable success by applying Mr. Hickox' interpolated curves, Figs. 19 and 20. Fig. 22 shows a section of

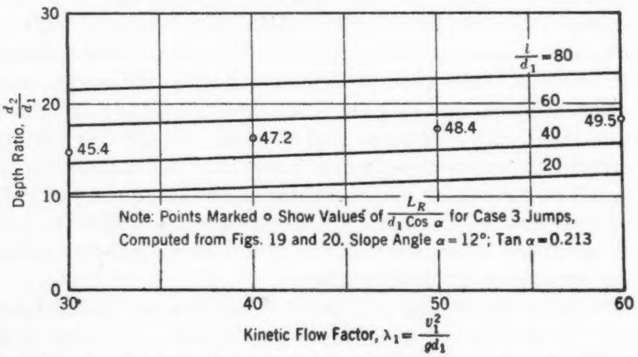

FIG. 22.—COMPARISON OF B. D. RINDLAUB'S DATA ON FIG. 10 WITH INTERPOLATED CURVES BY G. H. HICKOX

Fig. 10, comparing Colonel Rindlaub's data with computed points based on the Hickox curves. This verification of Mr. Hickox' interpolated curves is simultaneously a gratifying verification of the writer's analysis, since it indicates that the end of the roller is a satisfactory criterion for the location of the section of minimum depth for d_2.

Mr. Hickox' experiments on a steeper slope, and his attempt to develop a complete family of characteristic curves by interpolation of experimental results, are valuable contributions to the paper. The results of his computations are generally satisfactory when compared to the existing experimental data.

There is a danger in writing an equation for ϕ such as Eq. 19*b* or Mr. Hickox' equation in Fig. 18(*a*),

$$\phi = 1.49 - 0.00338\,\lambda \qquad (61)$$

There appears to be no reason why the relation $\phi = f_1(\lambda_1)$, Eq. 19*a*, should define a straight line. Although a straight-line equation does fit the data

within the limits tested, the curves so defined should not be extended indefinitely.

Simplified Analyses.—Mr. Kennison and Mr. Stevens propose simplified general analyses of the jump on sloping floors. Both suggest the assumption of a straight-line profile between sections 1 and 2 to evaluate the volume of the jump body. This idea was explored by Mr. Yarnell in 1934 and by the writer in 1939. In both cases, the results of the simplifications were disappointing when computed data were compared with experimental measurements over a full range of λ-values. The explanation is probably suggested by drawing a straight line from the surface at section 1 to the surface at section 2 on the measured profiles shown in Fig. 1. The discrepancy thus illustrated is considerably greater than that shown by Mr. Kennison in Fig. 11.

Mr. Kennison objects to the various manipulations involved in the derivation of Eq. 14. The derivation is not particularly complicated; in fact, the equations were expressly set up in a manner to facilitate algebraic solution, and the result is an equation for d_2 quite similar in structure to the formula for the level-floor jump. For a full range of values of λ, the results from Eq. 14 are much more accurate than those to be expected from Eq. 27*b*. Considering the nature of the phenomenon and the difficulties of observation, the range of observational errors, as computed by Professor Posey, is not excessive. Average curves drawn from the experimental data are believed to be of sufficient accuracy for practical use.

Mr. Stevens' generalized analysis is based not only on the assumption of a straight-line profile of the jump, but also on the aforementioned erroneous assumption concerning the length of the jump. Using Professor Ivanchenko's formula for the length of a jump on a level floor, Mr. Stevens derives an equation for the jump on floors of any slope, in which the length factor becomes quite important. Thus, using $m = \frac{9.3}{k^{0.185}}$, he discovered a zone of "fictitious" jumps which might have been considerably different had he used another arbitrary assumption for the length term. Following this conclusion, Mr. Stevens constructed several diagrams showing zones of fictitious, doubtful, and possible jumps. From Fig. 13(*a*), it appears that all of the jumps taken from Mr. Yarnell's experiments on the 1-on-6 slope are fictitious, since values of k in every case are less than 30. When these data are plotted on a curve similar to Fig. 14, however, only about half of the observed jumps appear to be fictitious. This comparison is shown on Fig. 23. The manner in which a fictitious jump differs from a real jump is not clearly shown. Really fictitious is Mr. Stevens' case of a hydraulic jump on adverse slopes. A stable jump cannot exist on an adverse slope except, possibly, when the slope is relatively close to zero.[33]

In the third paragraph following Eq. 36, Mr. Stevens states:

> "Analogous limitations exist in Eq. 14 from which it is seen that d_2 becomes infinite for $2\,\phi \tan \alpha = 1$ or for $(5.16 - 0.042\,\lambda) \tan \alpha = 1$ (Fig. 6(*a*)). This limitation will not apply to the 1-on-6 slope for which the author's formula is designed."

[33] "Fluid Mechanics for Hydraulic Engineers," by Hunter Rouse, McGraw-Hill Book Co., Inc., New York, N. Y., 1938, p. 301.

This statement is misleading, for, as previously noted, the algebraic expression for ϕ, Eq. 19*b*, is an equation of the curve in Fig. 6(*a*), derived from experiments in a channel with an average slope of 0.173. For this case, as observed by Mr. Stevens, a zone of fictitious jumps could not exist. The experimental data presented by Mr. Hickox subsequent to Mr. Stevens' discussion show that, similarly, for a slope of 0.336 a zone of fictitious jumps could not exist. It may be concluded, therefore, that, for every other slope, experiments will yield a separate curve of the ϕ-relation for "real" hydraulic jumps.

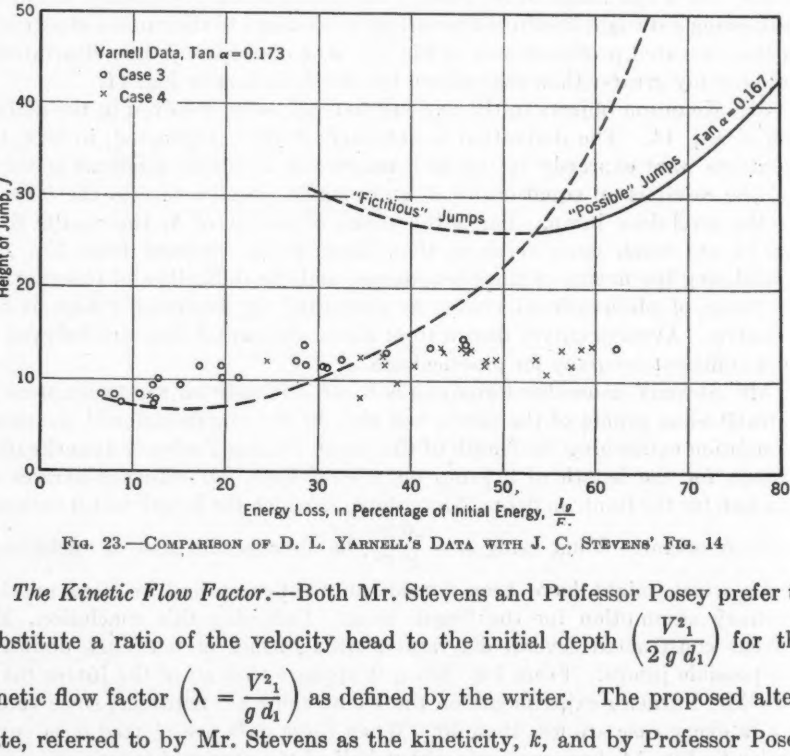

FIG. 23.—COMPARISON OF D. L. YARNELL'S DATA WITH J. C. STEVENS' FIG. 14

The Kinetic Flow Factor.—Both Mr. Stevens and Professor Posey prefer to substitute a ratio of the velocity head to the initial depth $\left(\frac{V^2_1}{2\,g\,d_1}\right)$ for the kinetic flow factor $\left(\lambda = \frac{V^2_1}{g\,d_1}\right)$ as defined by the writer. The proposed alternate, referred to by Mr. Stevens as the kineticity, *k*, and by Professor Posey as the velocity-head ratio, ω, is equal to $\frac{\lambda}{2}$. In recent years there has been a growing tendency to express the characteristics of all open-channel flow phenomena, including the jump, in terms of the general criterion of dynamic similarity, the Froude number, $\mathbf{F} = \frac{V_1}{\sqrt{g\,d_1}}$. For practical purposes, any of the three ratios, λ, $\mathbf{F}$, or *k* or ω, is applicable. The choice of definition appears to be a matter of personal preference. The writer objects to Professor Posey's term "velocity-head ratio," because it might be confused with a ratio of corresponding velocity heads in a model and its prototype. An advantage shared by both λ and $\mathbf{F}$ is that these ratios are equal to 1.0 for the condition of critical

flow. Thus, for λ or **F** greater than 1.0, the flow is "rapid," and, for λ or **F** less than 1.0, the flow is "tranquil." A significant advantage of the Froude number is that it expresses the ratio of the initial velocity of flow to the celerity of propagation of a gravity wave. The Froude number has the additional merit of being widely accepted as a criterion for similarity of related phenomena. The writer, like so many who have followed the work of Professor Bakhmeteff, will probably persist in the use of λ as defined in this paper. The most logical alternative, in his opinion, is the use of the Froude number.

Energy of Flow on Steep Slopes.—In discussing the general problem of the energy of flow on steep slopes, Mr. Fee develops the concept of a "mean energy line," which is frequently misunderstood. Referring to Eq. 54, he notes a discrepancy in the cited derivation[7] and requests further reference to the work of Harald Lauffer. The discrepancy shown in Eq. 55, copied from the text cited, is distinctly a typographic error. A translation of Mr. Lauffer's work, on which the development was based, was published by the Society in 1937.[34]

Acknowledgments.—The writer is indebted to the discussers of this paper, especially to Mr. Hickox for his information on the 1-on-3 slope and his assistance in preparing Fig. 23.

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[7] "Applied Fluid Mechanics," by Morrough P. O'Brien and George H. Hickox, McGraw-Hill Book Co., Inc., New York, N. Y., p. 293.

[34] "Abridged Translations of Hydraulic Papers," *Proceedings*, Am. Soc. C. E., November, 1937.

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DISCUSSIONS

SLUDGE DRYING DEVELOPMENTS AT CHICAGO, ILL.

Discussion

BY LLOYD M. JOHNSON

LLOYD M. JOHNSON,[6] ESQ.[6a]—Mr. Gordon has properly indicated that the problem of ash handling and disposal is minimized when fertilizer is produced. There is a further benefit at the Chicago Southwest plant in that the slag formation on the wall and roof tubes in the furnaces is held at a minimum. In burning a material like sludge with a high percentage of ash (compared with coal), the amount of slag is an important operating factor.

Although, essentially, operation appears automatic in a plant combining steam generation and sludge disposal, the need for properly trained and competent operators cannot be overstressed. The balancing of the various elements introduced in this combination results in a more delicate plant to operate than a straight steam generation plant.

Mr. Hansen notes the economy of handling partly digested sludge, where the treatment is sedimentation alone. Sludge from such a plant has a limited market as fertilizer, except in wartime, whereas with activated sludge treatment the economics may be different. The possibility of recovery of sludge gas for development of power is important.

Mr. Pearse has added data on procedures and operating results. In addition, the recent sale of grease-containing scum from a pumping station tributary to the Southwest works proves worth while at least during these times. Under a 2-yr contract, the Sanitary District receives 1.10 ¢ per lb for the scum at point of skimming. The contractor furnishes all labor for skimming and loading.

NOTE.—This paper by Lloyd M. Johnson was published in September, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1943, by C. W. Gordon; October, 1943, by Paul Hansen; and February, 1944, by Langdon Pearse.

[6] Commissioner of Streets and Electricity, City of Chicago, Chicago, Ill.

[6a] Received by the Secretary March 16, 1944.

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DISCUSSIONS

RELATION OF UNDISTURBED SAMPLING TO LABORATORY TESTING

Discussion

BY P. C. RUTLEDGE

P. C. RUTLEDGE,[11] ASSOC. M. AM. SOC. C. E.[11a]—The several discussers of this paper have contributed significantly to an understanding of the effects of sample disturbance on the results of soil tests. The discussions constitute ample evidence to support the underlying premise of the paper—namely, that the results of soil tests cannot be used directly to determine the behavior of soil in nature. In most applications, unmodified test results lead to solutions that have little meaning in terms of the true behavior of the soil and may result in unsafe conclusions. This is a significant factor in the application of soil mechanics to practical problems. It is an important reason why such applications cannot be reduced to simple terms and formulas, and is a serious obstacle to understanding by the average engineer working with structural materials. The errors introduced by differences in behavior between tests on structural materials and the finished product are usually small compared with the commonly used factors of safety; and, therefore, engineers have come to accept unquestioningly the direct use of the results of simple tests in machine and structural design. This fortunate situation does not exist for soils because neither the condition of the test specimen nor the stress conditions within the specimen are like those in nature.

Sample Disturbance and Consolidation Tests.—Three effects of sample disturbance on the results of consolidation tests have been emphasized by the discussers: (*a*) The significance of concave virgin compression curves; (*b*) the determination and significance of the preconsolidation load; and (*c*) the location of the most probable natural compression curve. These will be considered in order.

NOTE.—This paper by P. C. Rutledge, Assoc. M. Am. Soc. C. E., was published in November, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1942, by Benjamin K. Hough, Jr., and F. M. Van Auken; March, 1943, by Jacob Feld; April, 1943, by Raymond F. Dawson, and Hamilton Gray; June, 1943, by Karl Terzaghi, and D. P. Krynine; and September, 1943, by D. M. Burmister.

[11] Prof., Civ. Eng., Technological Inst., Northwestern Univ., Evanston, Ill.

[11a] Received by the Secretary March 20, 1944.

Concave Virgin Compression Curves.—Major Hough cites consolidation test results for St. Lawrence River clays with an initial void ratio of 1.6 in which the semilogarithmic virgin compression curves were concave upward. The writer has observed the same type of curves for many soils with initial void ratios larger than 1.5 but only when the test specimens had undergone a minimum of disturbance. For good samples with large initial void ratios, such curves are probably the rule rather than the exception. Major Hough also questions the undisturbed condition of the Mexico City clay which was used by the writer as an example. The samples were taken under the supervision of A. E. Cummings, M. Am. Soc. C. E., and J. A. Cuevas. Mr. Cummings arranged the transportation of the samples to the laboratory at Harvard University, Cambridge, Mass. Every precaution was observed to protect the samples and, as received in the laboratory, this clay had a rubbery consistency similar to artgum. It was not sensitive to shock or vibrations and was difficult to remold. The character of the clay, the fact that the laboratory preconsolidation load is within 3% of the existing pressure on the clay, and the sharp curvature of the compression diagram at the preconsolidation load—all indicate that this clay had suffered a minimum of disturbance, in spite of its extreme natural void ratio of 14.0.

Professor Gray has presented a challenging hypothesis for the compression of a "composite" specimen consisting of an undisturbed core surrounded by a disturbed or remolded annulus. The compression curve for such a specimen could well be slightly concave upward in the semilogarithmic diagram. The writer has never observed curves of this type for specimens known to be partly disturbed, possibly because the annulus of extreme sample disturbance was always trimmed from the test specimens. Professor Gray's example emphasizes: (1) The necessity for trimming off the outer annulus of bore-hole samples; and (2) the dangers of concluding that concave virgin compression curves represent minimum sample disturbance when there is a change in the techniques of sampling and sample preparation.

Professor Burmister's hypothesis for successive stages of compression of undisturbed clay and for the differences between the compression of undisturbed and remolded clays is based on a visualization of a chain-like structure of clay grains in undisturbed soil. He assumes that this structure is modified but not destroyed by compression whereas remolding causes a homogeneous dispersed state of the soil grains. These assumptions lead to an explanation for concave virgin compression curves and to the conception that the compression curve for remolded clay at large loads may intersect and lie above that for the same soil undisturbed. There is justification for the conception of successive stages of compression. At very high loads as the void ratio approaches zero, there must be a change in phenomena because compression can no longer be accomplished by reduction in void space. The slope of the curve should finally become the bulk modulus of the mineral making up the clay grains. What actually happens to compression curves at pressures greater than 25 kg per sq cm and to their relation to the curves for remolded specimens is speculative. The writer is not familiar with any tests that have resulted in

either the joining or the intersection of the compression curves for undisturbed and remolded specimens.

The Preconsolidation Load.—Mr. Van Auken and Professor Dawson have shown the difficulties in determining the preconsolidation load from laboratory tests on clays that have been subjected to a complex geological history and on those that have been subjected to alternate shrinkage and swelling due to surface drying conditions. Mr. Van Auken recommends careful study of all geologic evidence. The value of study of all such information is unquestioned but Professor Terzaghi (31*a*)[11b] makes the restrained but pertinent statement: "* * * as a rule, geological evidence regarding the thickness of vanished strata leaves a wide margin for interpretation."

For "normally" consolidated clays, Professor Terzaghi recommends the use of the existing overburden pressure in place of the preconsolidation load determined from laboratory tests. It remains, of course, to determine whether or not any particular clay specimen is "normally" consolidated. Certainly expansive clays that have been subjected to forces of surface drying, for which Professor Dawson has drawn curves, are not normally consolidated even though they have never been subjected to an overburden greater than that existing at the time of sampling..

The writer is indebted to Professor Terzaghi for clarifying the historical background of knowledge of the effects of disturbance due to sampling on the test behavior of clays. Professor Terzaghi has also succinctly differentiated between those parts of the writer's compression diagrams that are based on experimental evidence and those parts that consist of extrapolations based on assumptions which have not been proved. The distinction should have been stated more clearly in the paper.

Professor Terzaghi states that the actual condition of most clays during sampling and preparation for testing is one of practically constant void ratio while the externally applied pressure is reduced to zero by taking the sample out of the ground. Hence, the horizontal line is derived from the natural void ratio at the preconsolidation load to zero pressure in Figs. 8 and 14. The writer cannot agree, however, that this simplified picture, which does represent what actually happens, renders meaningless points A and B in Figs. 6 and 7 or the constructions whereby they were obtained. Most consolidation tests result in curves of the type of the heavy solid line curves CB′E in Figs. 6 and 7. Point B is located by comparing the shape of the initial part of the test curve with laboratory recompression curves that follow rebounds from known laboratory preconsolidation loads. Hence, the significance of point B can be stated briefly as follows: Had the clay been preconsolidated under conditions identical to those of the laboratory test, point B represents approximately the pressure and void ratio that the clay would have attained during its preconsolidation. Therefore, the pressure at point B should be called the "laboratory preconsolidation load" because it is dependent on laboratory conditions of compression.

[11b] Numerals in parentheses, thus: (31*a*), refer to corresponding items in the Bibliography (see Appendix of the paper), and at the end of discussion in this issue.

The meaning of the void ratio at point B is less directly apparent. Sampling operations weaken the soil structure simultaneously with the removal of natural external pressure. The void ratio at point B is the laboratory equilibrium point that would have been obtained had the soil structure been weakened without removal of the external pressure. As the writer stated in reference to Fig. 7, an imagined rebound from the fictitious void ratio at point B gives a direct measure of capillary pressure in soil test specimens. The hypothesis has been advanced that capillary pressure replaces in full the natural external pressure when a soil sample is removed from the ground. Analyses based on the void ratio at point B indicate strongly that any such capillary pressure is always less than the external pressure and is usually small if not zero.

The laboratory preconsolidation load is derived exclusively from similarity to laboratory precompression and recompression. Any relation between it and the maximum load to which a clay was subjected in nature requires the assumption that natural compression is similar in character or effect to laboratory compression. This assumption is not entirely without empirical justification. An example is the Mexico City clay for which the test results, shown in Fig. 5, yield a laboratory preconsolidation load almost exactly equal to the known pressure acting on the clay. A second example is a Chicago clay sample, for which consolidation and triaxial compression test results are shown subsequently in Fig. 15. Comparisons of approximate overburden pressures and laboratory preconsolidation loads for these soils and other soft saturated clays which contained no visible evidences of drying are shown in Table 3.

TABLE 3.—COMPARISON OF OVERBURDEN PRESSURES AND LABORATORY PRECONSOLIDATION LOADS FOR SOFT SATURATED CLAYS

(All Samples Cut by Hand)

Location	Sample No.	Initial water content[a] (%)	Depth, in Feet		Load (Tons per Sq Ft)		
			From ground surface	From water table	Over-burden[b]	Laboratory Pre-consolidation	
						From	To
Mexico							
Plaza de la Reforma[c]	H28	460.0	24	14.5	0.97	0.95	1.05
Chicago, Ill.							
Peoria Street	H13E	45.5	22	12	0.95	0.8	1.0
South Wabash Street	H8B	35.6	35	21	1.30	0.8	1.2
South Canal Street	H13C	25.5	31	22	1.20	1.1	1.5
Elston Street[d]	H13A	25.9	24	18	1.25	1.4	1.6
North State Street	P9-3	28.0	34	22	1.30	1.2	1.4
South State Street	P9-5	31.9	25	13	1.10	0.9	1.1
Detroit, Mich.							
River Rouge[e]	P3-4	{38.7, 31.9}	35	23	1.50	1.5	1.7

[a] Percentage of dry weight. [b] Approximate pressure. [c] Natural void ratio, 14.0. [d] One foot below stiff yellow clay. [e] Two consolidation tests.

Unfortunately the data referred to in the preceding paragraph are for samples at single depths in the upper parts of different deep beds of clay. The data from tests on bore-hole samples taken throughout the full depth of thick

clay strata are not conclusive for the following reasons: (*a*) The soil pressure changes and mechanical sampling operations causing sample disturbance increase with the depth of sampling; and (*b*) increasing sample disturbance has a tendency to decrease the indicated laboratory preconsolidation load.

As a result of the several preceding considerations in regard to the laboratory preconsolidation load, this value, determined by extrapolation from the results of consolidation tests, should have definite physical meaning, and studies to ascertain correctly its relation to overburden pressure under varying conditions should be continued. For such research the best possible undisturbed samples are required.

Location of the Natural Compression Curve.—Much of the discussion regarding the most probable location of the natural compression curve for clays refers to the flat curve in Fig. 8 and also to curve ab in Fig. 14. This type of natural compression curve was proposed by Professor Terzaghi (10), who has presented strong arguments in its favor. The writer has not been entirely convinced, however, that water content and overburden pressure data are sufficient to establish the curve as empirical (as Professor Terzaghi states) rather than hypothetical. The effects of changes in either plasticity or natural water content or both on the laboratory compression characteristics of clays have not been sufficiently explored to constitute conclusive evidence in either direction.

Professor Krynine has argued in favor of the flat type of natural compression curve as opposed to curve (4) in Fig. 8. In the case of soft clay subjected to loads at least as great as those of a medium sized building, these arguments have been answered by Professor Terzaghi's discussion, in which observed agreements are cited between settlements and predictions based on curves similar to curve (4), Fig. 8.

An important utilization of simplified soil sampling and testing operations is the approximate method, described by Professor Terzaghi, for determining the natural compression of clay from the results of tests on remolded specimens. This method, published in 1929 (29), has not been widely known or used. It is an example of the "flashes of engineering intuition" to which Professor Terzaghi refers (31*b*). The writer has applied the method to the results of tests on remolded specimens of soils for which test results for undisturbed specimens were also available. Except in the case of highly compressible volcanic and organic clays, the results were practically identical with the best estimates resulting from tests on undisturbed specimens. The writer agrees with Professor Terzaghi's evaluation of uses of this method. In particular, it is less expensive and no less reliable than results obtained from tests on inferior "undisturbed" samples. When the size of a job justifies the expense, the additional information obtained from the best possible undisturbed samples is a sound investment both for obtaining practical results and for increasing existing knowledge about the behavior of clays.

Sample Disturbance and Strength Tests.—Professor Burmister advocates triaxial compression tests in preference to unconfined tests on the basis of an unknown capillary pressure acting on the clay in the latter. The writer cannot agree with the arguments presented for the following reasons:

1. The application of lateral pressure through a rubber membrane does not eliminate or make known the stresses in the soil pore water.

2. Analyses of consolidation test results, previously described, indicate that initial capillary pressures in soil samples are small. Before testing they will be increased only by drying or by expansive tendencies caused by evolution of gas.

3. During the test, changes in capillary pore-water pressure can be caused only by volume changes in the specimen. No conclusive evidence is available, but existing data indicate very little tendency toward volume change of saturated clay tested in unconfined compression.

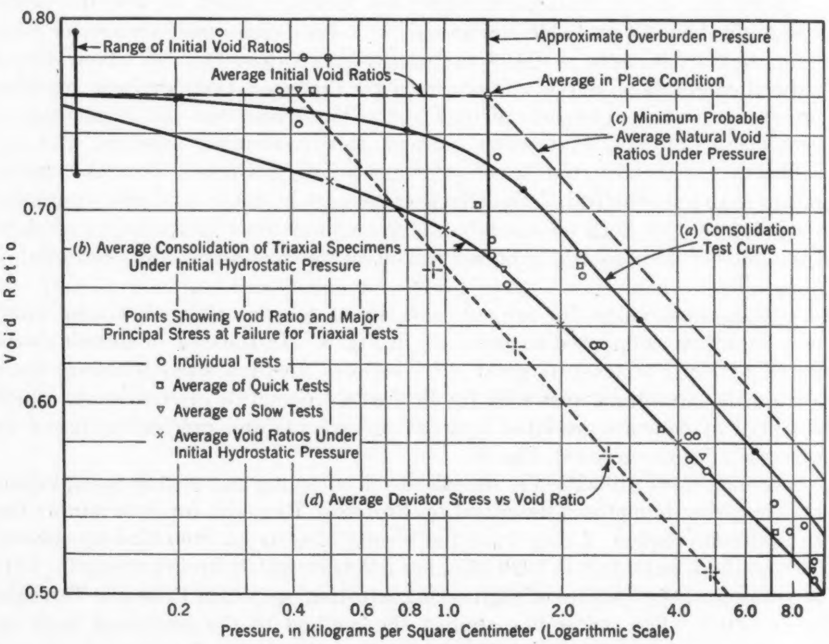

FIG. 15.—VOID RATIOS VERSUS MAJOR PRINCIPAL STRESSES

4. The unconfined compression test necessarily measures the applied vertical stress. This stress, when an unknown hydrostatic pore-water stress is considered, becomes the difference between major and minor principal stresses.

5. For a saturated plastic clay which does not change volume, the difference between principal stresses is substantially independent of small changes in the minor principal stress.

Mr. Van Auken and Professor Burmister both take exception to the writer's statements regarding the effects of volume change on triaxial compression tests of undisturbed natural clays. Both of them recommend preloading of triaxial compression test specimens by maintaining a constant ratio between major

and minor principal stresses until the major principal stress reaches some desired overburden pressure. The writer does not question the fact that clays consolidate both during triaxial compression tests and in nature but wishes to emphasize that, because of sample disturbance, the consolidation, the void ratios at which the clay is tested, and the pore-water pressures are completely different from those which will result in nature. The effects of sample disturbance on consolidation occur whether or not the test pressure conditions are like those in nature.

An example of the void-ratio changes during consolidated triaxial tests on soft clay is found in Table 2. The complete series of tests from which these data were abstracted was performed by H. H. Ku, Jun. Am. Soc. C. E. (38). In Fig. 15 the data are plotted in terms of void ratio. Curve (a) is the result of a consolidation test on one specimen from the cubic foot, hand-cut sample. The initial void ratio of the consolidation specimen is almost exactly the average of initial void ratios of all test specimens from this sample. Curve (b) is the average result of initial consolidation under hydrostatic pressure of all triaxial specimens. The maximum major principal stresses and the corresponding void ratios for individual tests and for averages of quick and slow tests are plotted as individual points. The consolidation time allowed in both the consolidation test and the preliminary consolidation of triaxial specimens was twenty-four hours, which was well beyond the theoretical 100% consolidation point for both. The maximum duration of the slow tests was also twenty-four hours.

Several facts should be noted in connection with Fig. 15:

(1) Curve (b) for consolidation under hydrostatic pressure falls below curve (a) for the consolidation test, indicating either that the preparation of triaxial specimens caused greater sample disturbance or that hydrostatic loading caused greater compression than consolidation test loading;

(2) The breaks in the compression curves (a) and (b) occur at almost the same stress and indicate the same laboratory preconsolidation load;

(3) The average final points for slow tests fall close to or on curve (b), signifying that consolidation is influenced primarily by the major principal stress; and

(4) None of the individual or average final test points fall definitely below curve (b), which indicates that shear (the maximum strain in all triaxial specimens was 20% or more) does not cause consolidation in excess of that to be expected under the major principal stress.

With the exception of the unknown cause for deviation under hydrostatic loading, consolidation under triaxial test conditions is found to be parallel to that in the consolidation test. Therefore, curve (c) was plotted representing the minimum probable natural void ratios that would occur in nature under stress conditions similar to those in the triaxial tests. In the tests, both the initial and final void ratios are much smaller than the corresponding values on curve (c) although the final values of the quick (20-min duration) tests approach curve (c).

Two options are now available for the evaluation of these triaxial compression tests: (*a*) The results can be corrected to a condition of zero pore-water pressure; or (*b*) the results can be corrected to the probable minimum void ratios which will exist in nature. The methods of correction are as follows: Under option (*a*) the maximum major principal stress is compared with the pressure required to produce the same void ratio in a comparable consolidation test and the difference is assumed to be pore-water pressure. Under option (*b*) the void ratio for any chosen minor principal stress is determined from curve (c), Fig. 15, and the difference between major and minor principal stresses corresponding to this void ratio is determined from the curve of deviator stress versus void ratio that is also plotted in Fig. 15. Both methods require radical assumptions. The first method assumes that consolidation under triaxial compression is exactly similar to that in consolidation tests. The data in support of this assumption have previously been presented. The second method assumes that all increase in strength is caused by decrease in void ratio.

The results of these two types of corrections are shown in Fig. 16. Corrections of the first type were made for each of the five series of individual tests. The total average correction line is also plotted for each of the five series. The close agreement supports this method for correcting for pore-water pressure. Corrections of the second type were made only for the average test results. There is a wide difference between the results of the two methods of correction. The evidence available (for example, the data cited by Major Hough) seems in favor of corrections by the second method although it is far from complete and correct. Therefore, the writer concludes that neither the strengths nor the pore-water pressures obtained from consolidated triaxial tests on undisturbed natural clays are representative of field conditions and that the results are unsafe when considering actual conditions.

It should not be concluded from these remarks that the triaxial compression test is without value. On the contrary, it is a tool of great usefulness. Even for plastic, saturated, undisturbed clays, testing in triaxial compression under small lateral pressures may be advisable for convenience in making the necessary test measurements. Another factor (which most of the discussers have agreed is important) that was not considered in the preceding analysis is the direct effect of sample disturbance on soil strength.

In connection with the effects of void-ratio changes, Professor Krynine's discussion of applications to the Coulomb formula conveys the impression that assumption (1) of a zero angle of internal friction for plastic clays and assumption (2) of void-ratio changes as a primary factor in observed strengths in triaxial tests are unrelated and have opposing results. This is not the case. Assumption (2) simply presents a means of explaining the results of triaxial compression tests in terms of assumption (1). Assumption (1) is based primarily on the results of field observations. As Major Hough has stated and the preceding analyses indicate, assumption (1) is difficult to substantiate by means of laboratory tests.

Summary.—Professor Gray's extension of the "chain" analogy to the solution of foundation problems appeals particularly to the writer. The clear statements of the limitations in present knowledge, contained in his first four

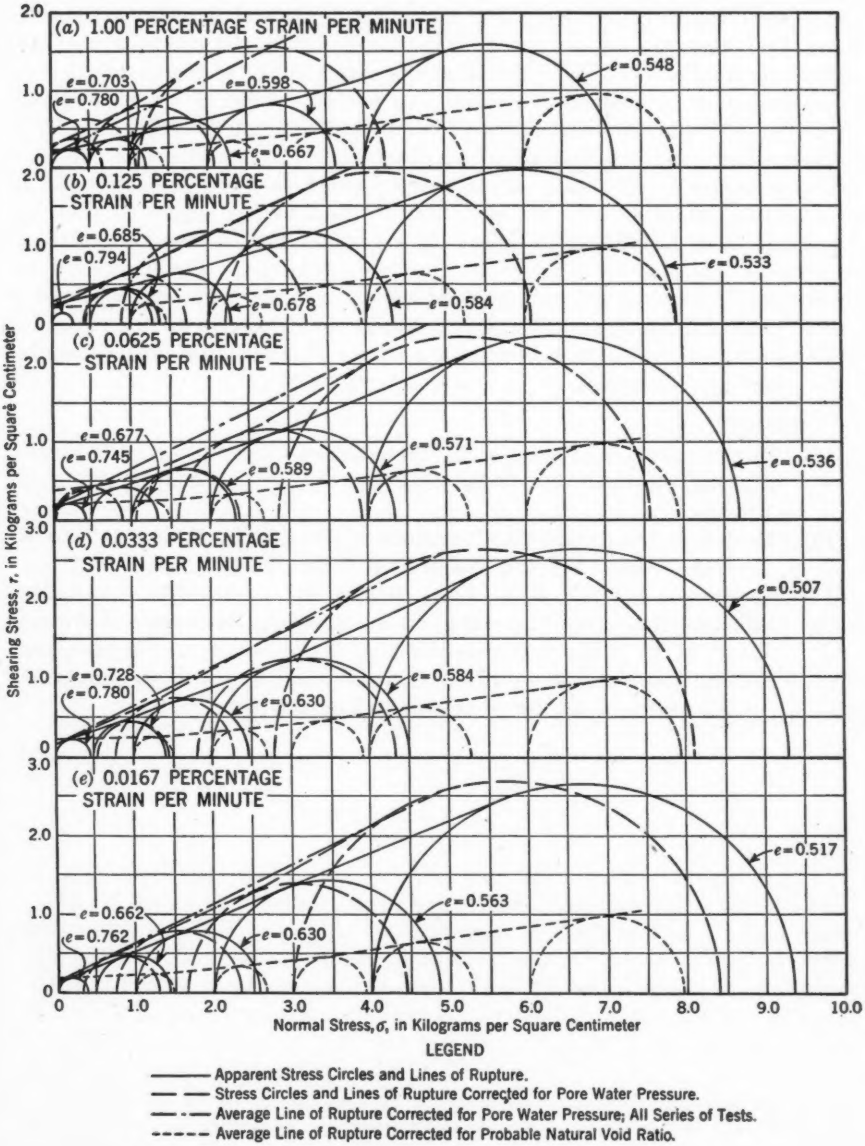

FIG. 16.—FAILURE DIAGRAMS—TESTS ON UNDISTURBED CHICAGO SOFT CLAY

(All Tests on Specimens Cut from 12-In. Cubical Sample with Average Initial Void Ratio = 0.759. Void Ratios Shown Are for Maximum Stress)

paragraphs, cannot be improved. The writer wishes to re-emphasize the one statement that individual cases of numerical agreement between predictions and field observations do not constitute proof that the methods used are generally applicable. The validity of the several links of the chain (that is, sampling, testing, analysis of stresses, and prediction) can be established only through (*a*) the accumulation of large numbers of complete field and laboratory observations; and (*b*) analysis of all observations by coordinating agencies. Such a program will require the concerted effort of every one interested in foundation problems. In this connection Mr. Feld has suggested that the methods of extrapolation described by the writer may be used to establish similarity between soils of known and unknown behavior. The suggestion has merit but is perhaps too optimistic an adoption of the methods, considering their present limitations. The writer thanks the discussers of this paper for their contributions to the subject of the effects of sample disturbance and for the stimulus of their ideas.

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(10) **"Undisturbed Clay Samples and Undisturbed Clays,"** by Karl Terzaghi, *Journal*, Boston Soc. of Civ. Engrs., Vol. 28, No. 3, July, 1941, pp. 211–231.

(29) **"Settlement Analysis—The Backbone of Foundation Research,"** by K. Terzaghi, *Paper No. 337*, World Eng. Cong., Tokyo, Japan, 1929.

(31) *Proceedings*, Am. Soc. C. E., June, 1943, p. 932. (*a*) p. 938. (*b*) p. 933.

(38) **"Effect of Time Rate of Loading on the Shearing Resistance of Natural Clay,"** by H. H. Ku, a thesis submitted to the Faculty of Purdue Univ., Lafayette, Ind., in June, 1941, in partial fulfilment of the requirements for the degree of Master of Science in Engineering.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

RIGIDITY AND AERODYNAMIC STABILITY OF SUSPENSION BRIDGES

Discussion

BY J. M. ROBERTSON, AND C. H. GRONQUIST

J. M. ROBERTSON,[30] JUN. AM. SOC. C. E.[30a]—Until the phenomenon of the aerodynamic action of suspension bridges is understood in its fundamental aspects, attempts to prevent the destructive vibrations of such structures are just as apt to make the situation worse as better. This paper represents a considerable advance toward the ultimate solution. Structural engineers must now be familiar with aerodynamics and vibrations—subjects they did not need in the past.

Referring to Eq. 29*a*, the logarithmic decrement δ is only approximately equal to $\frac{1}{2}\frac{\Delta W}{W}$. On the basis of the usual assumptions as to the nature of W, the correct relation between the two expressions is:

$$\frac{\Delta W}{W} = 2\,\delta\,(1 + \delta + \tfrac{2}{3}\,\delta^2 + \cdots) \quad (76)$$

Only for small values of δ is the relation used by the author accurate.

In the sentence following Eq. 34 the author states that "Accurate information on the structural damping of suspension bridges is lacking." This observation does not necessarily apply only to suspension bridges, as few data seem to be available on the damping of any except the very simplest of structures. Some data are available on the damping contributed by the material of which the structure is composed. An attempt will be made to summarize some of these data in this discussion.

Structural damping consists of at least two parts: The damping of the material in the structure created by the varying stress in the material; and the damping resulting from the comparative looseness of the joints in the structure. Little is known even as to the relative amount of damping at-

NOTE.—This paper by D. B. Steinman was published in November, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1944, by Edward Adams Richardson; and March, 1944, by R. K. Bernhard.

[30] Asst. Prof., Dept. of Eng. Mechanics, Pennsylvania State College, State College, Pa.

[30a] Received by the Secretary March 24, 1944.

tributable to each of these sources. In some vibrating machine structures, estimates of the damping effect of the material have indicated that in one case it was about 33% and in another 75%. Obviously, if the damping necessary to help keep the stress amplitudes in the structure within safe limits does not come from one source or the other the structure may fail at once or through fatigue. Few data are available on the damping effect of even the simplest of structural joints. O. Föppl[31] presents some test results in which the damping in a simple riveted joint was found to be about twice that in a solid member of similar strength. A welded structure, therefore, would have less damping effect than a riveted structure, at the same vibration amplitude.

The damping capacity of materials has been a subject of investigation since the days of Lord Kelvin. Hundreds of papers have been written on it, or on internal friction, or on "mechanical hysteresis," as it is sometimes called. Nevertheless, much information is still lacking on the damping capacity of engineering materials in the range of stresses of structural significance. Damping capacity is generally assumed independent of the frequency of vibration and has been shown to be a function of the stress or strain amplitude. As noted by the author, J. P. Den Hartog states[32] that the energy loss per cycle due to the damping in the material varies as $(a_m)^{2.3}$ "based on some experiments on soft steel in the dim past." Other writers have been more definite in indicating their supposed source for this relation. W. A. Tuplin,[33] for instance, attributes it to F. E. Rowett.[34] However, in his paper, Mr. Rowett noted that the loss per cycle varied as the third power of the stress amplitude and his data do not seem to indicate the 2.3-power law. The research of S. F. Dorey[35] among others has also verified this third-power variation for various steels. At Pennsylvania State College, in State College, the writer and A. J. Yorgiadis have also verified the third-power variation of damping capacity with stress amplitude for a considerable number of metals and plastics (not published).

Data on the variation of the damping capacity of steels ΔW_{sm}, energy loss per unit volume per cycle, with stress amplitude S_m, are shown in Fig. 14. Mr. Rowett's data were obtained on tubular specimens in torsion through static tests of great sensitivity and dynamic tests (by the decaying vibrations method) on a hard drawn (curve 2, Fig. 14) and an annealed mild (curve 1, Fig. 14) steel. Mr. Dorey's data (curves 3, Fig. 14) were obtained on solid cylindrical specimens in torsion by a static test similar to that by Mr. Rowett. Mr. Dorey studied steels of various compositions as may be noted from the key in Fig. 14. The data observed by the writer and Mr. Yorgiadis on a mild steel, obtained by the method of resonant vibrations using apparatus developed

[31] "The Practical Importance of the Damping Capacity of Metals, Especially Steels," by O. Föppl, *The Journal of the Iron and Steel Institute*, Vol. CXXXIV (1936), pp. 393–455.

[32] "Mechanical Vibrations," by J. P. Den Hartog, McGraw-Hill Book Co., Inc., New York, N. Y., 2d Ed., 1940, p. 254.

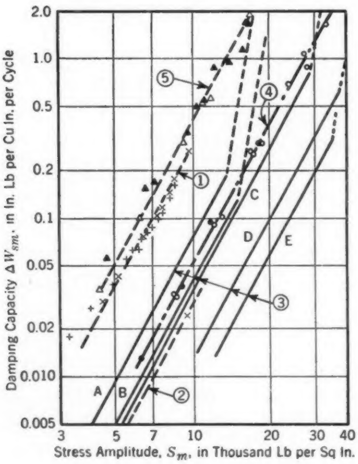
[33] "Torsional Vibration," by W. A. Tuplin, John Wiley & Sons, Inc., New York, N. Y., 1934, p. 205.

[34] "Elastic Hysteresis in Steel," by F. E. Rowett, *Proceedings*, Royal Soc. of London, Vol. 89, Series A, 1914, pp. 528–543.

[35] "Elastic Hysteresis in Crank-Shaft Steels," by S. F. Dorey, *Proceedings*, Inst. Mech. Engrs., Vol. 123, 1932, pp. 479–535.

by B. J. Lazan,[36] are also shown. The same material, a normalized mild steel tubing, was tested in torsion (curve 4, Fig. 14) and in direct, tension-compression vibrations (curve 5, Fig. 14). For all the data shown there is no evidence of a frequency effect at least in the range of frequencies covered. Up to a certain stress amplitude (curves 3, Fig. 14) the variation of ΔW_{sm} with $S^3{}_m$ is apparent.

The damping that occurs in solid specimens vibrating in torsion is much less than that in tubular specimens of the same material for the same maximum stress amplitude, due to the variation of the stress in the solid specimen. As



KEY

Curve (1) – F. E. Rowett; 0.17% Carbon and 0.24% Manganese; Annealed:

Symbol	Description of Test	Shearing Yield Stress (Kips per Sq In.)
×	Static	12.5
+	At 67 Cycles per Sec	12.5

Curve (2) – F. E. Rowett; 0.17% Carbon and 0.24% Manganese; Hard Drawn:

×	Static	31.0
-----	At 67 Cycles per Sec	31.0

Curve (3) – S. F. Dorey; Solid Cylindrical Specimens:

A	0.21% Carbon	24.2
B	0.30% Carbon	29.4
C	3.00% Nickel	47.2
D	Nickel Chrome	66.6
E	Chrome Vanadium	72.8

Curve (4) – J. M. Robertson and A. J. Yargiadis, S.A.E. 1025, Mild Steel, Normalized; Torsion Vibrations:

△	At 9 Cycles per Sec	22.0
▲	At 19 Cycles per Sec	22.0

Curve (5) – J. M. Robertson and A. J. Yargiadis, S.A.E. 1025, Mild Steel, Normalized; Direct Vibrations:

•	At 48 Cycles per Sec	64.0
∘	At 68 Cycles per Sec	64.0

FIG. 14.

shown by F. M. Lewis in his discussion of the Dorey paper,[35] and by W. Ker Wilson,[37] it is possible to relate the damping in solid and in tubular members under torsion if the energy loss is proportional to some definite power of the stress amplitude. For the third-power law, this indicates that the damping in the solid member should be two fifths that in a tubular member for the same stress amplitude in the outer fibers. If curve 3A, Fig. 14 (for a mild steel), is raised by the inverse of this ratio, there is a rough agreement with the other data given in Fig. 14 for mild steel tubular specimens in torsion. The type of stress as well as its magnitude has considerable effect on the damping capacity as may be seen from the data obtained by the writer and Mr. Yorgiadis on a mild steel (Society of Automotive Engineers (SAE) Specification 1025) under shear stresses and direct, tension-compression stresses. The damping under shear stressing (torsional vibrations) is about seven times that under direct stress. In addition to the variation in damping capacity with the foregoing factors, Fig. 14 indicates the influence of the nature and condition of the

[36] "Some Mechanical Properties of Plastics and Metals Under Sustained Vibrations," by B. J. Lazan, *Transactions*, A. S. M. E., Vol. 65, 1943, pp. 87–104.

[37] "Practical Solution of Torsional Vibration Problems," by W. Ker Wilson, John Wiley & Sons, Inc., New York, N. Y., 2d Ed., 1941, Vol. 2, p. 64.

material. For the mild steel tested by Mr. Rowett, annealing seems to have increased the damping (at the same stress amplitude) by a factor of about ten. Curves 3, Fig. 14, indicate a variation as great as ten times in the damping with the different compositions of steel. Apparently the higher the strength of the steel, the lower will be its damping capacity.

The damping capacity of steel is also a function of the previous stress history of the material. Continued cyclic stressing may cause an increase or decrease in the damping, depending upon the magnitude of the stress amplitude. This "cold working," as it is sometimes called, has been noted by Messrs. Dorey, Föppl, Lazan, and others, but there is little complete information on it. The most comprehensive study of this phenomenon for any material appears to have been made by A. U. Kutsay and Mr. Yorgiadis,[38] who noted a decrease as great as ten times in the damping capacity of a magnesium alloy as the result of cyclic "cold working." It is rather doubtful that one can expect steel to be affected quite this much, but it is well known to be affected.

Since considerable damping capacity is necessary to prevent excessive vibration amplitudes on a bridge subjected to vibratory forces, from the data given herein a low-strength steel with a high damping capacity may be superior to a high-strength steel with a low damping capacity. In Europe a considerable number of railroad bridge failures was attributed to the substitution of a high-strength nickel alloy steel of low damping capacity for an ordinary mild steel of higher damping capacity.

C. H. Gronquist,[39] Assoc. M. Am. Soc. C. E.[39a]—The coefficient of rigidity K in Table 2 is a measure of the resistance of the main span to sine-curve deflection under antisymmetrical sine-curve loading ($n = 2$) that is positive in one half span and negative in the other half span; the quarter-point deflection in either half span under such loading is p/K. It is of interest to compare this deflection with that for antisymmetrical uniform loading ($n = 2$) for which the resulting deflection may be obtained readily by use of a principle demonstrated elsewhere by the writer.[40]

That principle may be stated as follows: The deflection at the midpoint of a uniformly loaded simple beam, or at the midpoint of any uniformly loaded beam segment of constant moment of inertia between points of contraflexure with zero deflection, is equal to the bending moment at the midpoint divided by the buckling load of the beam, for elastic-curve deflection, as a strut of that length. For the loading considered, since the center of the main suspension span is a point of contraflexure with zero deflection and, moreover, since the value of the live load H is zero, each half span may be regarded as a simple beam under the applied uniform load p. The initial, elastic theory bending moment at the quarter point, consequently, is $\frac{p\,l^2}{32}$. The final buckling load

[38] "On the Torsional Damping Capacity of Solid Magnesium-Alloy Rods as Affected by Cold-Working," by A. U. Kutsay and A. J. Yorgiadis, *Journal of the Aeronautical Sciences*, Vol. 10, 1943, pp. 303–310.

[39] Associate Engr., Robinson & Steinman, New York, N. Y.

[39a] Received by the Secretary March 30, 1944.

[40] Discussion by C. H. Gronquist of "Rainbow Arch Bridge over Niagara Gorge: A Symposium." *Proceedings*, Am. Soc. C. E., April, 1944, p. 608.

for the suspension bridge is equal to the initial buckling load of the girder increased by the amount of the horizontal component of cable stress, in contrast to the buckling load for the arch, which is decreased by the amount of the thrust. The final value of the quarter-point deflection of the suspension span for this antisymmetrical loading (including the effect of deflection) is then:

$$\eta = \frac{M_e}{P_f} = \frac{\frac{p\,l^2}{32}}{4 \times 9.6\,\frac{E\,I}{l^2} + H_w} = \frac{p}{\frac{32}{l^2}\left(H_w + 4 \times 9.6\,\frac{E\,I}{l^2}\right)} = \frac{p}{K_u} \quad \ldots (77)$$

in which the amount of the horizontal component of cable stress remains unchanged, since the live load H remains zero, even considering the effect of deflection.

The stiffness constant for uniform antisymmetrical loading is practically the same as that of the author for sine-curve loading, except that the multiplier is 32 for the former as compared with $4\,\pi^2$ for the latter. This is in conformance with the greater deflection produced by the increase in load for the uniform load condition. A stiffness constant such as the one of Eq. 77, which is a measure of the resistance of the suspension bridge to deflection under antisymmetrical uniform load, was developed by the writer and later proposed for comparing the stiffness of suspension bridges in connection with a discussion on the stiffness of the self-anchored suspension bridge.[41]

The properties of the stiffness constant or final buckling load of the suspension bridge, with a two-hinged stiffening girder, for antisymmetrical loading, yield extremely simple data for computing the ratio of the final bending moment or deflection by the "deflection theory" to the moment or deflection by the usual elastic theory. The final moment is equal to the initial moment decreased by $(H_w + H)\,\eta$, thus:

$$M_f = M_e - H_w\,\eta = M_e\left(1 - \frac{H_w}{P_f}\right) = \frac{P_e}{P_f} \times M_e = R \times M_e \quad \ldots (78)$$

in which P_e is the initial buckling load of the girder, as by the elastic theory, unassisted by the cable pull. Such correction of the initial moment or deflection, by considering the effect of deflection, is analogous to that for a strut subjected to a lateral load or to initial deflection in addition to the direct loading. Formulas similar to Eq. 78 have been derived for the compression member,[42,43] and are closely applicable to the case of the suspension bridge, when it is considered that the direct load is tensile.

The girder rigidity ratio R in Table 2, or $\frac{P_e}{P_f}$, therefore, is a close approximation of the ratio of the final moments and deflections in the stiffening girder to moments and deflections for antisymmetrical loading computed by the elastic theory. The ratio may be taken as practically constant over the

[41] "Simplified Theory of the Self-Anchored Suspension Bridge," by C. H. Gronquist, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 989.

[42] "Buckling of Elastic Structures," by H. M. Westergaard, *ibid.*, Vol. LXXXV (1922), p. 576.

[43] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York, N. Y., 1936.

length of the span, and it will change little for different loadings producing maximum live-load moments at various points along the span. The higher value of the ratio for the maximum center moment is accounted for by a reduction in the distance between points of contraflexure. A practically perfect check is obtained between the values of R in Table 2 and the Baker constant[44] for the ratio of the deflection theory to the elastic theory for moments. By including the value of the live load H for any particular loading in computing the final buckling load P_f, a closer approximation of the moment or deflection ratio will result.

Deflection at the quarter points of the main span of a two-hinged girder suspension bridge produced by antisymmetrical concentrations at the quarter points is equal to 0.8 of the elastic theory moment under the load divided by the final buckling load, as for a simple beam with a central concentration. The final moment is:

$$M_f = M_e - H_w \eta = M_e \left(1 - \frac{0.8\, H_w}{P_f} \right) = \left(\frac{P_e + 0.2\, H_w}{P_f} \right) \times M_e \quad \text{..(79)}$$

indicating that the deflection effect for concentrated loads is less than that for distributed loading. The ratio derived in Eq. 79 for antisymmetrical concentrations may be applied approximately to any concentrated loading.

The foregoing formulas for deflections, stiffness constants, and deflection theory corrections, as derived for antisymmetrical main span loading, are applicable to the side spans of the suspension bridge, for which antisymmetrical full span loading ($n_1 = \pm 1$) is critical, when l_1 is substituted for $l/2$ in the formulas.

[44] "Suspension Bridge Analysis by the Exact Method Simplified by Knowledge of Its Relations to the Approximate Method," by Arvid H. Baker, *Engineering and Science Series No. 24*, Rensselaer Polytechnic Inst., Troy, N. Y., 1928.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

A METHOD OF COMPUTING URBAN RUNOFF

Discussion

BY W. I. HICKS

W. I. HICKS,[17] ESQ.[17a]—The discussion of this paper includes commendation, requests for additional information and data, and constructive criticism. The writer is grateful for the commendation and will answer the queries and criticism to the best of his ability.

Mr. Jarvis has noted certain divergences from agreement of the data plotted in Fig. 12. The writer selected data from seven drainage areas covering a wide range of urban development. As noted in the fifth paragraph of Part IX, there were irregularities in the records for Stations 4, 10, and 11, and Block B, which prevented full analysis of the peak runoff rates.

The record of Station 4, abscissa 1.50 and ordinate 0.80, is found on recheck of computations to be 1.33 and 0.80. As this drainage area is subject to heavy street flow at runoff rates greater than 0.45 in. per hr and to overflow from the drainage area at more than 0.60 in. per hr, agreement of record and computed flow would have been closer if the drainage system had been fully adequate to handle the storm.

Several of the records of Block B are subject to correction for overlap of preceding storms; one of these is analyzed in Fig. 25 for the point with an abscissa of 0.36 and an ordinate of 0.76. The peak at 8:45 a.m. is the result of rainfall from about 8:36 a.m. to 8:45 a.m. ($t_c = 9.2$); the rain prior to 8:36 a.m. would have caused a peak, if the rainfall had ceased at that time, with approximate shape and position as delineated ($t_c = 12 \pm$). Although no conclusive evaluation can be placed on its value nor on its recession curve, since no peak actually was formed in the record, it is apparent that the computed peak is more nearly in agreement than the plotted point indicates.

Station 11 is subject to heavier losses than would normally be expected on impervious areas, which accounts in part for the excess of computed peaks over actual peaks for that station. Although it may have been an error to include these divergent records in Fig. 12, they were included as being a part

NOTE.—This paper by W. I. Hicks was published in April, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1943, by C. S. Jarvis, S. W. Jens, and W. W. Horner.

[17] Civ. Engr., Storm Drain Div., Bureau of Eng., Los Angeles, Calif.

[17a] Received by the Secretary April 10, 1944.

of the data; the writer considers that the computation method is substantiated by the predominance of points falling within the 20% tolerance zone and the possibility of explaining the divergence of some of the other points.

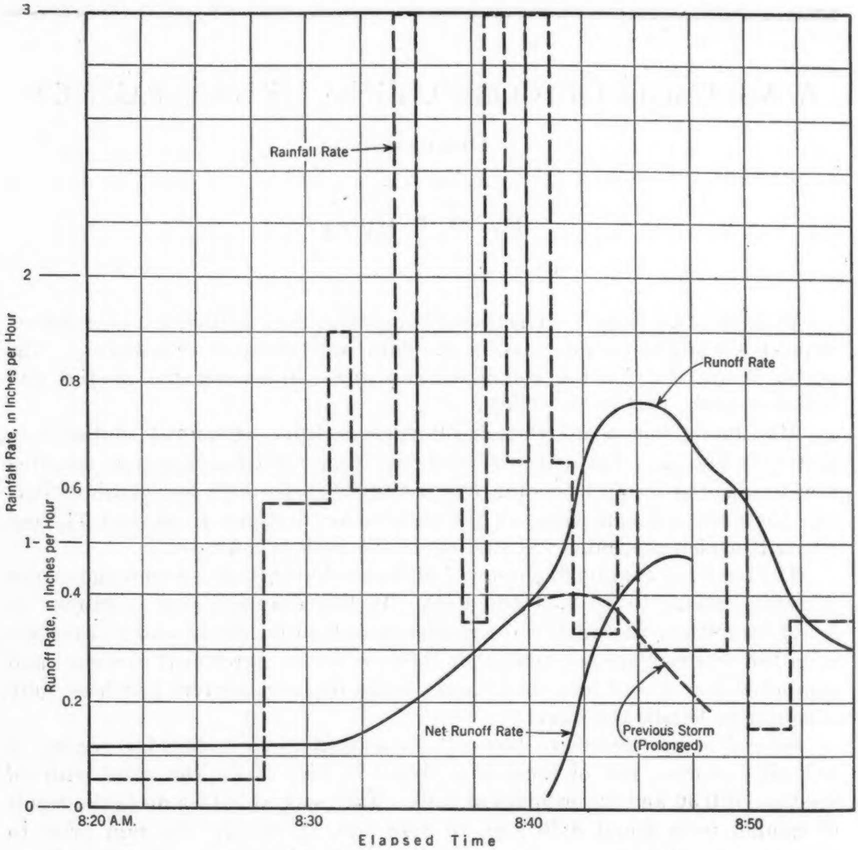

FIG. 25.—CORRECTION OF RUNOFF HYDROGRAPH FOR OVERLAP OF PRECEDING STORM; CITY BLOCK 4841, ST. LOUIS, MO., SEPTEMBER 28, 1919

In compliance with Mr. Jarvis' suggestion of the possibility of computing a value of C from the data in Table 12 in the Rational Formula,

$$C = \frac{q_1}{I} \qquad (10)$$

Table 17 was prepared. It is largely self-explanatory: Cols. 1 to 5, inclusive, are prepared as Mr. Jarvis suggests, and the remaining columns are devoted to computation of a coefficient C_1 modified by the moisture factor M and the shape factor F embodied in I_H (Col. 6, Table 17). The factor I_m (Col. 8) is the rainfall rate for the time of concentration (Col. 7) for the standard rainfall curve having an hourly intensity I_H. The modified C_1 is expressed in the

equation,

$$C_I = \frac{q_3}{I_m\, M^{A_{px}}} \quad \text{.....................} (11)$$

The divergence between average high and average low ratios of q_c/q_3 in Table 12 is from about 118% to 87%; that for C is from 137% to 72%; and that for C_1, from 118% to 85%. Thus, for an area of stated physical and rainfall characteristics, Mr. Jarvis is correct.

TABLE 17.—INVESTIGATION OF COEFFICIENTS OF RUNOFF, STATION 3

No. (1)	I (2)	q_1 (3)	C (4)	$\frac{C}{C_{avg}}$ (5)	I_H (6)	t_c (7)	I_m (8)	q_3 (9)	$M^{A_{px}}$ (10)	C_1 (11)	$\frac{C_1}{C_{1\,avg}}$ (12)
1	1.19	0.430	0.362	0.684	0.793	28.3	1.230	0.413	0.550	0.610	0.87
2	0.68	0.320	0.470	0.888	0.459	33.4	0.640	0.300	0.708	0.660	0.94
3	0.88	0.456	0.520	0.982	0.550	32.7	0.780	0.422	0.660	0.817	1.17
4	0.59	0.315	0.534	1.008	0.435	32.2	0.621	0.275	0.672	0.659	0.94
5	1.09	0.610	0.560	1.04	0.760	29.4	1.048	0.570	0.670	0.812	1.16
6	0.77	0.560	0.727	1.37	0.528	33.3	0.739	0.510	0.790	0.875	1.25
7	1.06	0.830	0.783	1.48	0.664	30.8	0.981	0.730	0.880	0.843	1.20
8	0.64	0.505	0.790	1.50	0.455	37.5	0.596	0.455	0.915	0.834	1.19
9	0.55	0.530	0.961	1.82	0.427	37.1	0.563	0.460	1.035	0.790	1.13
10	0.61	0.192	0.315	0.595	0.418	32.4	0.593	0.182	0.710	0.432	0.62
11	0.73	0.229	0.314	0.593	0.447	30.0	0.665	0.219	0.524	0.630	0.90
12	0.63	0.227	0.360	0.680	0.436	30.2	0.645	0.201	0.542	0.596	0.85
13	0.64	0.260	0.416	0.786	0.424	31.0	0.620	0.250	0.604	0.666	0.95
14	0.50	0.147	0.294	0.555	0.359	33.7	0.501	0.147	0.538	0.545	0.78
Averages (numerical).....			0.529	....	....	....	....	....	....	0.698	....
Averages (percentages):											
High values.........			137	....	....	....	....	....	....	118	....
Low values..........			72	....	....	....	....	....	....	85	....

However, to follow through with computation of runoff coefficients based on the ratio of observed runoff to observed rainfall without analysis and synthesis of the various factors affecting runoff, it will be necessary to gather and analyze data from a much greater number of drainage areas of different sizes, soils, and states of physical development. Although the possession of such data is greatly to be desired, it is doubtful whether personnel and money will be available to acquire it except through the passage of many years. In the meantime, the method of analysis and synthesis, refined and revised, as Mr. Jarvis suggests, by the accumulation of additional authentic data, holds forth the greater promise for current and future design. It is more flexible by reason of the use of the distribution factor (Fig. 5) and the factor for adjusting confluence of dissimilar areas (Fig. 14); when the designer has mastered its not too difficult method of operation, he will approach his problem with a clearer understanding of the fundamentals involved.

The area limitation in the paper has been set by the 120-min time of concentration set up in Figs. 5 and 16 and Table 7; larger areas can be handled by a reasonable extrapolation or by use of the Method of Summing Hydrographs.

In answer to Mr. Jens' questions on the factor M, the relations found by Eq. 5 led to the statistical analyses of rainfall data from the local Weather Bureau of the two composite factors, $I\,F$ and $I\,F\,M$. For loam or clay pervious areas, the maximum value of $I\,F\,M$ (not necessarily of $I\,F$) will

cause the maximum runoff. Therefore, the seasonal variation of M is embodied in the data and need not be considered further in design. These analyses were made for several time durations and it was found that the resultant M was slightly greater for the short durations. On account of the small variation, it was ignored for the sake of simplicity in computations. However, in the analyses of individual rainfall-runoff records, M was computed from the data for each storm.

The additional slope data requested by Mr. Jens for drainage areas in Table 1 are given in Table 18. It is true that lag time is dependent on these

TABLE 18.—SLOPES FOR DRAINAGE AREAS IN FIG. 1 (PERCENTAGES)

Station No.	Overland			Gutter			Drain		
	Minimum	Maximum	Average	Minimum	Maximum	Average	Minimum	Maximum	Average
3	0.6	50.0	15.0	0.50	22.0	4.25	0.75	8.51	1.036
4	0.6	10.0	2.6	0.25	7.50	1.61	0.33	2.67	0.946
10	1.75	6.00	2.42	1.13	1.13	1.13	....	....	....
11	1.27	6.30	2.70	0.20	2.00	0.92	....	....	....

slopes as well as on the shape of the drainage area and the distribution of impervious development therein. In Fig. 9(d), lag time is shown as a function of the time of concentration which in turn is a function of the slopes of the surfaces and the channels of flow. The shape of the area and the distribution of impervious development determine the point of the maximum quick concentration of runoff; the elapsed time between this point and the gaging station is the lag time.

It is gratifying to learn that Mr. Jens finds the straight-line relationship between volume of storage and rate of discharge in his study of open-channel recession curves. In Eq. 7, K corresponds to the inverse of $\frac{\Delta D}{\Delta q}$ in Figs. 8 and 9(a).

From a theoretical standpoint, Mr. Jens is correct in his contention that the loss of rainfall on an impervious surface is a fixed amount. On broad drainage areas, however, absolute imperviousness is a goal reached only in varying degrees of perfection. In determining the amount of loss of rainfall on impervious areas, data were used from impervious Stations 10 and 11, and from experiments on overland flow on impervious surfaces. In the case of Stations 10 and 11, analyses were made as in Figs. 6 and 7, and the amount of rainfall and of loss between control points was determined. The average rate of loss is plotted as an ordinate against the corresponding average rate of rainfall in Fig. 26. The results indicate an increase of loss rate with an increase of rainfall rate. The higher loss for Station 11 offers some possible explanation: Paving grades are flatter with a consequent wider spread of flow; the pavement is not fully impervious; numerous utility and sanitary sewer manholes furnish small avenues of escape; and sidewalks around basement openings and area lights allow leakage into the basements. These deductions are substantiated by the facts that street railways use ballast subdrains, that public utilities pump out their conduit systems and the city outfall sewer runs under heavy

pressure for an extended period after heavy rains, and that numerous buildings use basement pumps. The use of 10% instead of the higher percentages indicated in Fig. 26 was determined by the smaller percentage of loss in the

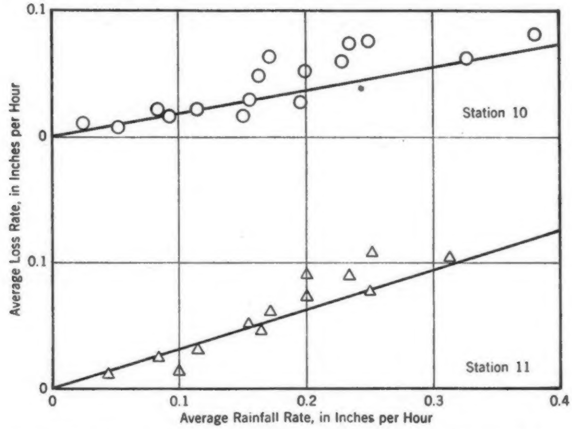

FIG. 26.—VARIATION OF LOSS RATES WITH RAINFALL RATES ON NOMINALLY IMPERVIOUS AREAS

overland flow experiments, the comparatively pervious soil under Stations 10 and 11, and the better surface drainage which an adequate storm drain system offers.

Mr. Jens' analysis of the storage equation is the most effective which the writer has encountered in that it avoids the tedious trial-and-error method so commonly used. Its use will furnish relief in reservoir problems where the inflow is translated to the gates and spillway of the dam by a high-velocity deep-water pressure wave. The complex pattern of velocities in the cross section of a conduit makes difficult the accurate determination of the changing shape of the hydrograph as it progresses downstream. In the studies leading to the method of peak-rate reduction used in this paper, investigations were made of a number of mathematical solutions of the storage equation; none seemed to fit the behavior of the hydrograph. The storage equation method, when applied to conduit flow, has the following inconsistencies: (*a*) It tends to shorten the near-peak-rate duration of flow instead of spreading the reduction in the vicinity of the peak; (*b*) computation for a 5-min reach of conduit shows much more than five times the reduction computed for a 1-min reach; and (*c*) the lag between the peaks of the inflow and the outflow hydrographs seems to bear no definite relationship to the time of flow in the reach of conduit selected. For example, in Fig. 23 the lag is about 5 min, whereas the time of flow is about 17 min in the 5,585-ft reach.

Hydrographs of storm drain flow, as gaged (Figs. 6 and 7) and as standardized for storm drain design (Fig. 15), are characterized by the sharp rate of rise on the accretion side of the curve and a comparatively short duration of near-peak rate, which duration increases with the time of concentration. These characteristics suggest a greater rate of peak-rate reduction for the slender hydrographs with small times of concentration than for the broader peaked, longer

time hydrographs. The volume of available storage in a conduit t_f minutes long is the product of the length and the cross-sectional area, or $60\ t_f\ V \times Q/V = 60\ t_f\ Q$ cubic feet (see step 3 under the heading in Part VII, "Reduction of Peak Rate"). Since this volume of storage is independent of the slope or velocity of the conduit, the percentage of peak-rate reduction, computed by any method, remains constant regardless of the slope or slopes of the conduit. Based on these considerations and on the fact that Eq. 4 was derived from a study of the elements of actual hydrographs, the method of computing peak-rate reduction used in this paper was adopted.

The method of reduction is based on a standard shaped hydrograph. If, in the process of summing, the resultant hydrograph becomes so irregular as to not conform in shape to a standard hydrograph of any time of concentration, then Mr. Jens' objection to a standard method of reduction is correct. However, if the time of concentration is large, the percentage indicated in Table 7 is so small that no great error will be incurred by its use.

In discussing Eq. 5, Mr. Horner has assumed that either the summation of hydrographs or Eq. 5 is used in design practice. Under the heading, "VIII. Methods and Form of Runoff Computation," the Peak Rate Method is recommended for use except for stated conditions which require the Method of Summing Hydrographs. Eq. 5 has a limited special use on areas for which rainfall and runoff gagings are available.

Symbol M is an empirical factor derived from studies of rainfall-runoff data on pervious and partly pervious areas for the purpose of gaging the effect of antecedent precipitation, wind, temperature, etc., on the loss of storm precipitation into the soil. The writer also noticed its resemblance to the M derived by Mr. Horner.[16] Analyses of its effect on runoff were made with Eq. 5 for areas having different percentages of pervious area. It was used in the form M^{A_p} because, as A_p approaches 0 for impervious areas, M^{A_p} approaches unity.

The factor E is introduced to weight M for the seasonal variation in the amount of soil moisture loss. It was derived arbitrarily as follows: (*a*) Evaporation quantities for Fallon, Nev., California, Ohio, and Kingsbury, Calif.,[18] were averaged and summed for the calendar year; (*b*) the summation was converted into percentages of the total for the year; (*c*) for dates at regular intervals, the percentages covering 60 days antecedent were tabulated and plotted as ordinates on abscissas of the dates; and (*d*) these percentages, increased by a uniform 10%, were subtracted from 100% to form the E-curve.

In regard to the time required for design procedure by this method, the writer has encountered the same objections mentioned by Mr. Horner. In practice, the new method requires no more time than the Rational Method when applied to the routine peak-rate design which constitutes the bulk of municipal work, and it offers a comparatively simple method of solving special problems, such as retarding reservoirs, etc.

As the method of design in this paper was formulated for the solution of metropolitan Los Angeles drainage problems, informal discussion has raised

[16] "Rôle of the Land During Flood Periods," by W. W. Horner, *Proceedings*, Am. Soc. C. E., May, 1943, p. 665.

[18] "Handbook of Hydraulics for the Solution of Hydraulic Problems," by Horace Williams King, 3d Ed., McGraw-Hill Book Co., Inc., New York, N. Y., 1939, p. 444.

the question of its applicability to other localities. The method is based on the following local fundamental data:

A. Rainfall
   1. Intensity-Duration Curves (Fig. 1)
   2. Variation of Rainfall Volume with Location (Fig. 2)
   3. Pattern of Major Storms (Fig. 3)
   4. Antecedent Precipitation (Moisture Factor M) (Table 5)
   5. Rainfall Distribution Factor (Fig. 5)

B. Soil Losses (Fig. 4)

C. Classification of Drainage Areas (Table 6)
   1. Percentages of A_d, A_i, A_p.
   2. Lengths and Slopes for Overland Flow
   3. Typical Block Dimensions

D. Climatic Conditions
   1. Temperature
   2. Humidity
   3. Frozen Ground—Snow
   4. Dry Weather Soil Cracking

Since these local data are used in constructing the table of basic peak runoff rates (Table 7), the charts of runoff factors (Fig. 13), and other tables and charts used in design, it is apparent that changes in the basic data would involve varying degrees of change in such tables and charts. The process of developing a runoff design method in another locality would be to analyze the data for that locality, derive the fundamental information indicated by the numbered figures and tables in the foregoing outline, and, by the methods of this paper and the previous paper by Messrs. Horner and Jens,[4] construct new tables and charts for design.

In other localities, use may be made of the charts and tables of this paper for peak runoff rates (not necessarily for shape of hydrograph, which is dependent on storm pattern) from areas having negligible or no percentages of pervious area (impervious business and manufacturing, apartment house, and bungalow court classifications, Table 6). As it is possible that intensity-duration curves for other localities will not have the same shape and pattern as those in Fig. 1, it will be necessary to use the following equation to secure the hourly rainfall rate for determining the runoff factor (Fig. 13):

$$\frac{I \text{ (for stated } t_c \text{ from their local rainfall curve)}}{I \text{ (for stated } t_c \text{ from 10-yr curve, Fig. 1)}} \times 1.01 = I_H \ldots\ldots (12)$$

The value of I_H will be amended by the distribution factor (Fig. 5), if the area exceeds 100 acres, before it is used to determine the runoff factor. The product of the runoff factor and the basic peak runoff rate (Cols. A, Table 7) is the peak rate of runoff, in cubic feet per second per acre. Because of possible variation of the shape of the intensity-duration curve and of the rainfall distribution factor, the solution will be subject to some error.

[4] "Surface Runoff Determination from Rainfall Without Using Coefficients," by W. W. Horner and S. W. Jens, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 1039.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

RAINBOW ARCH BRIDGE OVER NIAGARA GORGE

A SYMPOSIUM

Discussion

BY M. HIRSCHTHAL

M. HIRSCHTHAL,[53] M. AM. SOC. C. E.[53a]—This Symposium on a long span arch bridge is naturally of great interest to engineers, particularly to those concerned with the design of railroad or highway bridges. The profession is grateful to Mr. Hardesty, et al., for the detailed presentation and analysis of the unusual erection and construction problems and their ingenious solution for the Rainbow Arch Bridge over the Niagara gorge from its very inception to its completion. The reason for some of the omissions and the inclusion of some of the material in the Symposium is not clear, however, in view of the thoroughness of the treatment of the entire project.

For instance, there is no information on the various considerations that led to the selection of the particular span length. On the other hand, the model tests seem to have been made on a two-hinged arch to corroborate its rejection rather than on a model of a hingeless arch to verify the stresses and deformations or strains found in the design of the structure selected—the one that was actually built.

Although model tests are very informative, it is dangerous to base all conclusions on them as being applicable quantitatively to prototypes, especially those of extraordinary size. The interest in the Rainbow Arch Bridge is (as has been true in the case of other bridges) due to its unusual span length—the "longest in the world." Had an arch of moderate span length been planned for this site, the writer doubts greatly whether model tests would have been made. Arch action is a normal structural phenomenon that occurs in many forms, although it still has some unexplained features, such as results of temperature

NOTE.—This Symposium was published in October, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1943, by Egidio O. Di Genova, and Charles Mackintosh; January, 1944, by C. M. Goodrich, and T. Kennard Thomson; February, 1944, by C. M. Spofford, L. J. Mensch, and O. H. Ammann; March, 1944, by Louis Balog, William G. Rapp, and Leon Beskin; and April, 1944, by Neil Van Eenam, and C. H. Gronquist.

[53] Concrete Engr., D. L. & W. R. R., Hoboken, N. J.

[53a] Received by the Secretary March 25, 1944.

effects which model tests (in addition to any mechanical movement of one support) could elucidate. However, even in arch action tests quantitative application of results to prototypes would have to be made with caution.

Since model tests had been made, it would have been exceedingly interesting to have had comparative results of tests on two models, one a fixed-end model and the other the two-hinged model described in the Symposium. Both, of course, should be of identical dimensions, so that resulting deformations for various conditions of loading could be compared. There is no doubt that the very magnitude of the structure produces problems in design that may readily be ignored in smaller structures. For these reasons, results from scale models, particularly when they are of different material from that of the prototype, must not be the sole basis of decision. A 5-ft model is slightly longer than $\frac{1}{200}$th of the span length of the prototype, and, since the deflection varies with the cube of the span for a scaled load, the slightest error will be enormously magnified in the study of deflections.

Moreover, the writer questions the original selection of a two-hinged (flexible) arch and all the preliminary work attached to it. After complete analysis and detailed estimates were made, this type of design was discarded in favor of the hingeless arch (fixed) because of the economy of the latter. With outcrops of rock on either side of the gorge or waterway available for abutment foundations, a hingeless or fixed arch would suggest itself immediately as the logical and most economical type. This selection would be discarded only if it should prove too expensive.

In the consideration of economy, the writer is inclined to believe that the selection of three (or possibly even two pairs) of ribs would have resulted in some lessening of duplication of sections but would have reduced girder sections for the floor beams and floor system, thus causing a reduction in the dead load and obviating the necessity of using any alloy steel in the superstructure. This arrangement, moreover, would have required the selected arch rib section for one rib only (if three ribs were used), whereas the side ribs could have had carbon steel sections, while a stiffer cross section of structure would have been provided. The study of the deformation of the hingeless arch should have led to the selection of a section of rib varying from crown to abutment to provide a deeper section at approximately the quarter point. This would have reduced greatly the deformations due to live load in the vicinity of the quarter point even though this varying section would have complicated the detailing somewhat.

Because the only partial live load considered in the arch analysis of the Symposium was that on half the span, it might be interesting to note that the specifications of the American Railway Engineering Association[54] for Masonry Arches for Railroad Loading require the following live-load conditions:

1. For maximum positive moment at the crown, assume the middle quarter of the span loaded.

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[54] *Manual for Railway Engineering*, A.R.E.A., 1943, Committee 8, Masonry, Specification 312, Section 10, p. 8–61.

2. For maximum negative moment at the crown, assume all of the span loaded except the middle quarter.

3. For the maximum positive moment at the springing line, assume five eighths of the span loaded from the opposite end.

4. For maximum negative moment at the springing line, assume the adjacent three eighths of the span loaded.

A structure as important as the Rainbow Arch Bridge should be designed for at least the maximum live-load specifications of HS20 trucks, as defined by the American Association of State Highway Officials.[55] The Veterans' Gorge Arch Bridge in Rochester, N. Y., was designed (in 1930) for 30-ton truck loads to anticipate future increase in truck loading. Moving artillery over the highways has raised the question of rating highway bridges to permit the passage of guns. The increase in live load would have added comparatively little to the total load and resulting stresses and therefore to the cost of the structure, but would have provided for future loading increases.

As for methods of design, it has hitherto been acceptable practice to design arches by the "elastic theory" (an unfortunate expression, as the "theory" is not elastic). In this Symposium, however, doubt is cast on the "assumptions" underlying this theory and a deflection "theory" or method is presented as being exact.

The cables of a suspension bridge form an inverted arch or catenary and are flexible to such an extent that a stiffening truss is introduced in order to minimize the deformations. The resultant action is a pull or tension exerted by the cables under load, in contrast to the thrust or compression resulting from the loads in the case of the arch. In both cases bending moments are caused by the eccentricity of the applied forces. However, an arch (steel or concrete) has relatively much greater stiffness without considering the stiffening effect of the spandrel system (generally ignored in arch design) to resist deformation or distortion.

All theories applied to structures are based on assumptions, and no assumptions are absolutely exact. The closeness of the results of the proposed to those in the actual structure will depend on the closeness of the assumption to actual conditions. No assumption is made in the "Theory of the Elastic Arch" (so-called elastic theory) which is unsound or questionable, particularly for a steel structure in which the modulus of elasticity changes but very slightly in the various sections of the arch. In fact, the only assumptions other than that of elasticity made in this type of analysis for the fixed arch (aside from that of uniformity of *E*) are that the supports shall be unyielding, so that the horizontal span length remains constant, and that there will be no vertical displacement of one abutment with respect to the other. For the Rainbow arch, with the two abutments fixed in sound rock, the assumptions are as closely approximated as possible. The resulting thrusts will result in an infinitesimal yielding of the abutments or of the rock foundation, but such yielding would be microscopic.

[55] Standard Specifications for Highway Bridges, A.A.S.H.O., 2d Ed., 1935, Div. V, Section 2, Paragraph 7a.

Hardy Cross, M. Am. Soc. C. E.,[56] showed that differences in the value of E in the various sections of such structures were not serious in affecting the analysis of the arch. He investigated effects of variations as great as one third and one half the values of E. Such large variations within the range of working stresses are unusual. In the light of recent research developments, control over concrete will be more strictly observed in the future, and greater uniformity in the product and therefore in the value of E may be expected.

In the early stages of concrete construction for railroad structures in the United States, the arch type of design was quite popular. When the New Jersey cutoff of the Delaware, Lackawanna and Western Railroad Company was projected (1907–1911) between Hopatcong, N. J., and Slateford, Pa., of a total of seventy-six bridges and culverts included in the construction of the line, sixty (that is, 80%) were of the concrete arch type. In the construction of the Pennsylvania cutoff of the same railroad (1912–1915) between Clarks Summit, Pa., and Halstead, Pa., there were sixty-six arch structures, inclusive of two viaducts. One of the viaducts, the Tunkhannock Viaduct, is almost half a mile long, and includes the maximum span length (180 ft clear, or 208 ft center to center of piers), but this structure, of course, was designed for railroad engine loading.

Although considerably greater span lengths have been used for highway loading in the United States, the greatest extension of span length of concrete arches has been developed on the European continent, but no structure has approached the proportions of the Rainbow Arch Bridge.

Naturally, the thought would occur to a designer of concrete bridges: What could be done in concrete for a site like the Rainbow gorge? Fig. 43, for

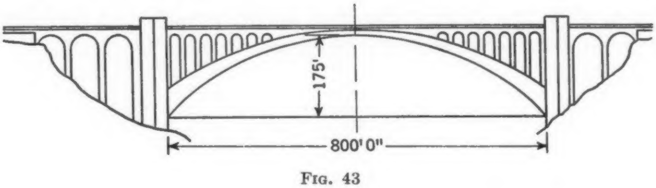

FIG. 43

example, seems a rather startling extension of span length for a concrete arch and approaches closely the limits visualized by E. Freyssinet.[57] The writer, when designing the Veterans' Gorge Bridge over the Genesee gorge in Rochester, proposed and made preliminary studies of a 600-ft arch for the main opening to span between the outcrops of rock on either side of the gorge. The resulting structure, however, did not meet the esthetic requirements of the architects retained for the work, so that the 300-ft clear semicircular arch was adopted and built as part of the viaduct. Based on 5,000-lb concrete and intermediate grade reinforcing steel, with working stresses of 1,500 lb per sq in. and 20,000 lb per sq in., respectively, the center rib crown section would have been 16 ft deep by 8 ft wide, 23 ft deep at the springing, and the reinforcement would con-

[56] "Dependability of the Theory of Concrete Arches," by Hardy Cross, *Bulletin No. 203*, Univ. of Illinois, Urbana, 1930.

[57] "Memoires de la Société des Ingénieurs Civils de France," by E. Freyssinet, July–August, 1930.

sist of 25, $1\frac{1}{2}$-in. square bars in the intrados and extrados. The floor system would have been a two-way reinforced concrete slab of 24-ft clear spans in both directions. The length of the span of the spandrel arches surmounting the main span would have been 24 ft.

The foregoing dimensions and sections are based on orthodox design. For economy of loads, the use of lightweight concrete, hollow arch ribs, and a larger percentage of alloy steel reinforcement would have to be investigated in an effort to reduce the dead load. In the concrete arch, dead loads result in stresses amounting to very nearly 50% of the total stress from all loads or forces and from the bending moments due to eccentricity of thrusts, including temperature effects and rib shortening. Incidentally, the field for the use of alloy steel for reinforcement of concrete has not yet been explored and may become of some importance in the future.

Of course, the construction or erection of such an arch structure would present a complicated set of problems, the solution of which would include the construction of a cableway and the erection of an "umbrella" section over each abutment for the support of steel arch ribs, as falsework for the concrete arch ribs. It would involve the shifting of the arch rib centers for the construction of the outside ribs and their dismantling—no inconsiderable task (much like the steel arch erection problem). The design of the long columns would be a special problem, as would the order in which the voussoir sections should be placed. All these are difficult but not insuperable problems. At the site of the Rainbow Arch such a proposed bridge would have been an interesting and instructive job.

Incidentally, the late William H. Burr, M. Am. Soc. C. E., proposed a 750-ft concrete arch to span the Harlem River at Spuyten Duyvil in New York, N. Y., some years ago, so that the span length proposed in this discussion is within range of a structure actually designed although not built.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

RÔLE OF THE LAND DURING FLOOD PERIODS

Discussion

By W. W. Horner

W. W. Horner,[35] M. Am. Soc. C. E.[35a]—The discussion of this paper is extremely interesting both with respect to the corroborative data presented and with respect to pertinent questions raised. Some of the discussers allowed their curiosity to carry them beyond the scope of the subject matter in the paper, and many of the questions relate to other phases of hydrology.

Mr. Kazmann suggests that the paper could be strengthened by including a treatment of the relation of infiltration to ground-water movement, particularly to changes in water table elevation. Such a study would undoubtedly have been extremely interesting. At one phase of the investigation the writer made a quick review of observation well data available at the Black Lands Experiment Station at Waco, and found that water table elevations could not be correlated satisfactorily with changes in infiltration capacity.

The writer's detailed presentation was restricted to surface runoff and related stream flow. The quantitative value of the infiltration capacity of the soil surface seems to be related to the rise and fall of the water table only when the water table is high enough to bring the surface soil within the capillary fringe. Undoubtedly, an investigation such as Mr. Kazmann suggests would be of real value to the general knowledge of hydrology, and possibly sufficient data are now available at Waco and Coshocton to justify such an undertaking.

Messrs. Hoyt and Langbein ask a number of questions, some of which are pertinent to the subject matter of the paper. These questions are discussed herein in the order of their appearance and with reference to the paragraphs.

In the seventh and eighth paragraphs Messrs. Hoyt and Langbein offer an expanded title for the paper, part of which is "Where Infiltrated Water Does Not Appear in the Stream as 'Quick-Subsurface Return Flow,'" and suggest that basic data might have been presented to prove that "subsurface inflow constitutes less than 10% of the flood volume." Both of the foregoing facts

Note.—This paper by W. W. Horner was published in May, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings* as follows: June, 1943, by L. K. Sherman; September, 1943, by Raphael G. Kazmann, and W. G. Hoyt and W. B. Langbein; October, 1943, by Charlie M. Moore; and December, 1943, by Waldo E. Smith, P. B. Rowe, and C. S. Jarvis.

[35] Cons. Engr. (Horner & Shifrin), St. Louis, Mo.

[35a] Received by the Secretary February 18, 1944.

were stated by the writer in the course of a general description of the basin; in the methodology used and described in the paper, ground water was separated out from the hydrographs of flood rises. An inspection of these hydrographs affords an approximate verification of the statements quoted.

The writer has no responsibility for the statement quoted by Messrs. Hoyt and Langbein.[19] Their interpretation of that statement might apply to parts of the Trinity basin outside the Blacklands, but not the East Fork above Rockwall.

In the ninth paragraph Messrs. Hoyt and Langbein make their own computation that 78% of the total runoff at Rockwall was surface runoff. The writer made no assumptions or computations of the percentage of total surface runoff at Rockwall. Surface runoff was determined for each rise of the hydrograph for the thirty-five storms through the elimination of base flow by generally accepted methods.

In the tenth paragraph Messrs. Hoyt and Langbein seem to question the results of the writer's presentation because it did not include research studies similar to those carried out at Coshocton. Undoubtedly such a research study can be made from data available at Waco; but it is entirely outside the scope of the subject matter presented.

In the eleventh paragraph the first question raised by the discussers, if properly related to the Trinity basin and to the material presented in the paper, is whether reduction in surface runoff can be assumed to be the equivalent of the reduction in flood runoff of the stream when there is no considerable subsurface return flow. With regard to the use of cultivated lands, if agriculturalists restricted their operations to "soils that combine high infiltration capacities with satisfactory moisture-retention properties," the nation would probably be on the verge of quite a famine.

The questions raised in the twelfth paragraph and thereafter are directly pertinent to the paper, but they also reflect the natural difficulty of fully understanding new methodologies and their implications until after a personal effort has been made to apply them. Messrs. Hoyt and Langbein state that they reviewed the writer's report as consultant to the Department of Agriculture, but apparently they did not distinguish between the two principal parts of the methodology as related to the objectives of that investigation. For each storm the report contained discussions of the probable reasons for differences between computed surface runoff and the surface runoff in the actual hydrograph. To clarify the natural resulting confusion, the following more extended discussion of Steps 7 and 8 is presented:

The hourly pattern of excess rainfall was computed for each rainfall station in the manner indicated in Table 3. These patterns then had to be transferred to patterns for each of the ten sub-basins. These in turn were weighted to produce the average hourly excess for the basin as a whole. The values of excess rainfall for most of the storms varied materially between rainfall stations. Many of the storms were actually partial-area storms. Initially the pattern

[19] *H. R. Doc. No. 708*, 77th Cong., 2d Session (letter from the Secretary of Agriculture transmitting a report of a survey of the Trinity River watershed based on an investigation authorized by the Flood Control Act of June 22, 1936).

transfer was made on the basis of Thiessen area weighting. Later, it was made on the basis of inverse distance ratios, which seemed more nearly to reflect the actual variation across the basin. The latter method was adopted for determination of the correction factors and gave reasonable results for all the more uniform storms.

For the more highly varying and for some of the partial-area storms, neither of these arithmetical procedures could give a satisfactory indication of the actual distribution of excess rainfall over the sub-areas, and for such conditions the drawing of isohyetal maps was essential. The use of such maps throughout the study would have appreciably increased the significance of the results.

In all cases, excess rainfall was computed initially and transferred to a mean value for the entire basin by the proportional method. It was then compared to the measured surface runoff as determined from the Rockwall gage. Although this gage was a chain gage read twice daily, and, although for one or two of the greater floods the record included estimated values of discharge, it was necessary to assume that the complete record was valid and it was so accepted as a basis for comparison.

If the computed excess rainfall was greatly different from the measured runoff, this fact was taken as a "red flag" and the possible reason for the difference was explored and discussed. The same procedure was followed if the later computed hydrograph was seriously out of phase with the recorded hydrograph. Restudies of this kind resulted in several revised computations for some of the storms—for example, such a comparison served to clarify ambiguities of dating the cooperative observers' rainfall reports. For some of the storms the discordance could be largely accounted for by the method of transfer of values to the sub-basins; in others, the drawing of isohyetal maps giving rational consideration to the distribution of rainfall between stations quite generally produced weighted values for the basin having closer accord with the measured values. For those storms in Table 4, which were selected for inclusion in Table 5, isohyetal maps were initially made up on the basis of rainfall information only. For use in the second steps, such maps were in some instances readjusted to produce excess rainfall exactly equal to the measured runoff. This procedure was in lieu of the use of a correction factor.

For example, for storm 2 under the initial procedure the weighted excess was 1.77 in., requiring a correction factor of 0.91, but the later computed hydrograph was somewhat out of phase. Thus, a revision of the interpretation of dates on certain ambiguous rainfall reports was indicated. On this basis the recomputed weighted excess was 1.83 in.; and the correction factor, 0.88. This factor is shown in Table 4 and is based on transference by isohyetal map. This second computation was not influenced by the measured amount of runoff. For the succeeding "before and after" study, the pattern was shifted to produce better conformance with the shape of the hydrograph.

Table 5 is the compilation for those storms where excess rainfall was computed on the basis of the *M*-curve and transferred to the sub-areas either by the proportional method or by isohyetal maps. The only influence of the known "measured" runoff on this procedure was the indication of the possibility of error in the proportional method of transference. If (as was later concluded)

such maps could have been used to advantage throughout this study, no reference whatever to measured values would have been involved.

A recheck of the data used in Table 5 discloses that one additional flood (No. 9) was excluded from this table besides those stated. Also one flood indicated in the range from 0.5 to 1.0 in., in the December 1 to March 15 period, was incorrectly included in that range. This table therefore is based on twenty-two floods for which the computation of excess rainfall and the indicated correction factors were completed before any comparison was made with measured runoff.

With respect to the question about the M-curve, the initial M-curve was developed from a study of the small watershed data from which the infiltration-capacity curves were derived.

Subsequent to the analysis of the thirty-five storms above Rockwall, determinations were made of the M-values (assuming all other data valid), the use of which would have resulted in reproducing the value of surface runoff taken from the hydrograph at Rockwall. This series of M-values departed somewhat from the M-curve shown in Fig. 4, placing this curve somewhat lower during the winter season and somewhat higher in July and August. Some composite of these two M-curves undoubtedly can now be used in this basin for forecasting the flood flow from any specific storm pattern.

Table 5 is the test of the ability of the described method to produce values of flood flow in fair conformance to measured values. Messrs. Hoyt and Langbein confuse the character of the data in Table 5 with the procedure in the succeeding stage. They ask, "If partial advantage is taken of the runoff as it was, why not take full advantage." This is exactly what was done in the second stage of the study in which the original computed values are adjusted by use of the correction factor or by revised or original isohyetal maps to an exact equality with the actual surface runoff. The reweighting of excess rainfall in accordance with the proposed program of land-use revision then produces "after" values of flood runoff. Thus, the present or "before" value having been adjusted to the measured value, the "after" value resulting from a computation identical in every respect (except for the land program) makes possible the determination of the reduction in flood flow which that land program would cause.

The writer is in full agreement with the questions concerning the use of hourly rainfall values. The procedure is quite sensitive to rainfall intensity. Mean rates for, say, 30 min could have been used to advantage if they had been generally available. It is interesting, however, that, even with only hourly values of rainfall derived from poor precipitation data, the results are generally consistent. In this investigation the objective is the determination of the average flood reduction with relation to flood magnitude. Possibly the poor rainfall data explain some of the scatter of points in Fig. 6; but nevertheless a rather significant trend is indicated.

In the fourth paragraph from the end of their discussion Messrs. Hoyt and Langbein state:

> "The report explains the influence of antecedent conditions as they affect soil moisture on the infiltration capacity during a storm. Increasing the volume of infiltration prior to a storm decreases the capacity of the basin to retain water. Therefore, under the changed conditions, to what extent would the presumed increase in infiltration operate adversely to lower infiltration capacities during subsequent storms?"

This question appears to be answered by the actual data presented. Infiltration-capacity curves, for each separate complex, were derived from small watershed data taken under a natural sequence of precipitation periods. Therefore the curves for pasture, for example, indicate the value of infiltration capacity for pasture when the soil moisture is related to antecedent infiltration for pasture. Since the infiltration for pasture is higher than for cultivated lands subjected to the same rains, the soil surface under pasture grass, and among other things protected from the effect of rain impact, seems to retain a higher capacity even though the moisture content be greater than that for adjacent cultivated fields.

The economic implications again are beyond the scope of the paper. However, the paper did not cover the subject of flow reduction as a whole, but only the "reduction in flood flow." Any adverse effect would seem to apply only to consumptive use of impounded flood waters. This, eventually, conceivably might be offset in some part by improvement in base flow.

Mr. Sherman's discussion presents the results of similar investigations in which he has used a methodology paralleling in many respect that which the writer applied to the Trinity River study. Mr. Sherman appears to be in thorough agreement with the practice of time condensation by which infiltration-capacity curves derived from natural rainfall data may be reduced to the approximate equivalent of curves that would have resulted from rainfall-intensity rates which exceeded infiltration-capacity rates in every case. This method permits a correlation between the infiltration-capacity curves derived from the two types of data. If such a condensation of the original derived curves is satisfactory, the condensed curves could, with some assurance, be expanded in the same manner, in order to be matched appropriately with any particular rainfall-intensity pattern.

Mr. Sherman also independently relates infiltration capacity to soil moisture through a compilation of antecedent infiltration. In his practice he has developed a somewhat simplified procedure as compared to the writer's *M*-curve, by relating infiltration-capacity values to three conditions of soil moisture deficiency.

It is of interest that, since the presentation of his discussion in May, 1943, Mr. Sherman has made a much more detailed analysis of the mechanics of infiltration and has developed a relationship between infiltration-capacity values and soil moisture deficiency as separated into capillary and noncapillary deficiency.[36]

As stated in the paper and in connection with the discussion by Messrs. Hoyt and Langbein, a second series of *M*-values was determined for the whole

[36] "Infiltration and the Physics of Soil Moisture," by L. K. Sherman, presented at the regional meeting of the Am. Geophysical Union in Berkeley, Calif., on February 15, 1944.

of the East Fork of the Trinity basin above Rockwall. On the basis of these additional values, a modified M-curve was developed. All of the data and the original and modified M-curves are shown in Fig. 10. The placing of the original M-curve, in the period from July to November, was based on a relatively few infiltrometer runs. Using the basin-wide data, satisfactory results cannot be secured from an infiltrometer during the period when these black

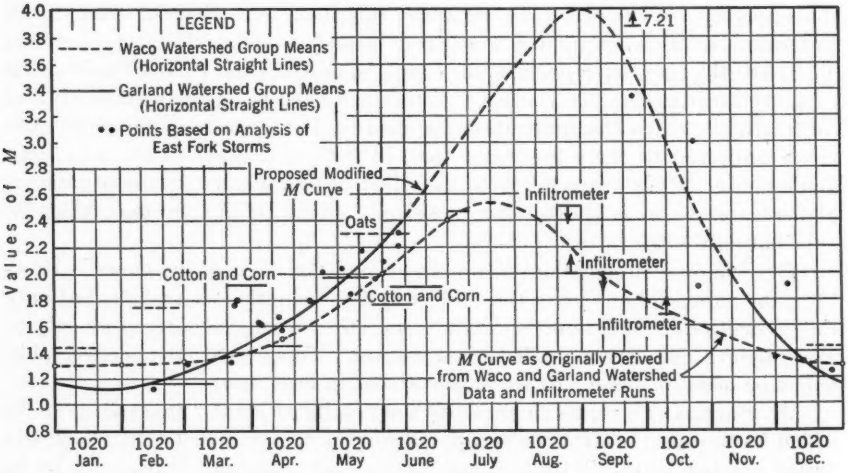

FIG. 10.—VARIATION, DURING ONE YEAR, OF THE VALUE OF M REQUIRED TO PRODUCE AN INFILTRATION CAPACITY OF 0.4 IN. PER HR

soils are extensively cracked. Therefore, probably much of the applied water leaves the area of the infiltrometer plot through wide cracks and supplies non-capillary deficiency over a large area around the plot.

An inspection of the modified M-curve, even though it is sketched in, indicates that M-values of between 3 and 4 in. would control surface runoff for the months of July to October, inclusive, and further indicates that the probability of a major flood during those months is extremely remote. For example, for normal precipitation conditions in the three months prior to the last 30 days before a storm, infiltration capacity would not drop to 0.4 in. per hr until either—

(*a*) Infiltration in the storm had exceeded about 3.5 in.; or

(*b*) Some combination as, for example, 4 in. of infiltration from the preceding 30 days plus 1.5 in. of infiltration in the specific storm period had occurred.

A study of the 5-yr record indicates a number of excessive precipitation periods that produced only insignificant rises of the hydrograph of stream flow. One such period involved about 7 in. of rainfall within 3 or 4 days.

Mr. Moore presents an interesting account of the difficulty in maintaining adequate supervision over a large group of rainfall gage observers. The writer made an analysis of some of the small watershed and runoff plot records at the Tyler Station, involving certain of the east Texas sandy soils. Infiltration

capacity was determined for these soils for a number of complexes but the results were not used in the Trinity River investigation, since it was found that a negligible percentage of the soils in the basin above Rockwall was of that type. Out of these studies, however, some tentative conclusions were reached. One of these was that the infiltration capacity of terraced watersheds was not appreciably higher than that for watersheds having the same type of cover but without such structures. The greatly increased infiltration in the terraced channels probably resulted in a generally higher moisture content which offset the other advantages of the treatment. Another interesting result of these studies was finding that the terraced watershed in Bermuda grass and another watershed in native protected forest rarely produced any surface runoff.

Mr. Moore refers to the use of the rational formula by the Operation Division of the Soil Conservation Service with the choice of a constant value of C for each land use and soil group. The procedure which the writer has outlined would permit the choice of a value of C for each such group. This would properly reflect maximum soil moisture conditions and consequently would probably produce maximum values of runoff. Such an application may be justified where the ultimate purpose is to design structures of adequate capacity, as, for example, outlet channels for terraced areas.

Mr. Smith again raises the question of the use of average hourly rainfall intensities and comments further on the unsatisfactory character of the data. Apparently he would like to see an investigation of this type in which all the desirable data are of a highly accurate character. Such a study would then fall into the field of a high-grade research project. The investigation reported by the writer was one which needed to be made and had to be made on the basis of the data available. The results appeared to be highly significant, probably more so than any one would have anticipated before the effort was undertaken. It is to be hoped that at some future time, when technical personnel again becomes available for extensive hydrologic research, the methodology may be applied to some such basin as that of the Licking River in Ohio.

Mr. Smith's presentation of the data in Fig. 9 is extremely interesting. The use of information of this kind when properly correlated with corresponding infiltration-capacity curves would probably permit further material improvement in controlling soil moisture factors such as the M-curve which the writer has used. Mr. Smith has performed a real service in bringing up to date a brief bibliography for deriving infiltration-capacity curves.

The writer is highly gratified with Mr. Rowe's discussion. In a first draft of the paper the writer made an attempt to include a discussion of the analysis of drainage basins in which stream flow was materially affected by "quick subsurface return flow." He reviewed some of the excellent information which has become available from the Southern Appalachian Experiment Station of the Forest Service, U. S. Department of Agriculture, but was unable to cite good examples of the use of these data in the quantitative determination of the relation of stream flow to land factors. For this reason, the writer found it necessary to restrict his detailed presentation of methodology to the situation in which stream flow resulted primarily from surface runoff and in which infiltra-

tion capacity of the soil surface was the primary control. Under the heading, "Subsurface Retention and Detention," the writer expressed the hope that information might be presented with respect to a satisfactory methodology in which quick subsurface return flow represented a material part of stream flow.

Mr. Rowe has described briefly the approach in California to an analysis of this latter situation. He states that infiltration-capacity curves are derived from small plot infiltrometer measurements. Such derivations must be used with extreme caution. The writer has seen infiltrometer runs made on shallow surface soils and steep slopes where no surface runoff whatever was produced, and yet the streams in this area were subject to floods of considerable magnitude. In one particular instance, further investigation indicated that subsurface flow was preceding down slope through the porous surface soil at extremely high rates and that this flow generally came to the surface at the margins or banks of the small streams or gullies. Recently the writer analyzed infiltrometer runs on a deep fill of clean sand with practically no lateral slope but with a water table 5 ft to 6 ft below the surface. An analysis of these runs showed that they made possible a determination of infiltration capacity for only a few minutes after surface runoff began. Thereafter a mounded water table was flushed to the surface and the infiltrometer record showed merely the rate at which subsurface water was transmitted away from the vicinity of the plot. Both of the foregoing are unquestionably extreme cases. Many situations have been recognized in which surface runoff was produced and controlled by the infiltration capacity of the soil, but at the same time infiltration capacities continued at high values, soil-storage capacity became filled, and subsurface return flow occurred at high rates. A methodology that would recognize all of these conditions and permit the reproduction of flood flow in the streams on the basis of precipitation intensity and land factors will be much more complex than that presented by the writer. Such methodologies have been devised in connection with certain flood control surveys of the U. S. Department of Agriculture. It is hoped that some of these may be presented in detail to the engineering profession.

The remarks of Mr. Jarvis are particularly interesting. The writer is in agreement with his conclusion 3. With respect to the fourth paragraph, the writer would also anticipate that, for a watershed such as the North Concho in Texas, a revised program of land use and management might well result in a greater reduction of flood flow than was found for the Trinity basin.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

STRENGTH OF SLENDER BEAMS

Discussion

By GEORGE WINTER

GEORGE WINTER,[8] ESQ.[8a]—The agreement among the discussers on the insufficiency, in some respects, of present design specifications for slender beams is gratifying to the writer. Messrs. Jasper, Wadleck, and Hussey also concur in recognizing the usefulness of this analytical treatment for establishing more rational design procedures than those used currently.

From the designer's point of view the practical conclusions drawn from the paper by Mr. Hussey, particularly Fig. 7, are most interesting. These conclusions, in which the writer fully concurs, are: (1) Current specifications do not account for the very high lateral strength of box beams, and, therefore, lead to uneconomical design of such sections; (2) it is safe to use I-beams with an L/b-ratio larger than 40, provided that working stresses are adjusted correspondingly; (3) present design formulas for I-beams are conservative; and (4) for practical use the values of the rather cumbersome secant formula should be approximated by some more convenient expression of the Johnson or Rankine type, in much the same way as it is done in some current column specifications.

Fig. 7 demonstrates that present design stipulations are about correct or just slightly conservative for laterally weak members, such as 18-in., 54.7-lb I-beams. These specifications are extremely conservative, however, for laterally strong sections, such as 15-in., 75-lb I-beams. For example, for $L/b = 40$, Fig. 7 shows that for the latter beam a design stress is justified which is more than 30% higher than that for the former. This demonstrates the practical importance of one of the principal findings of this investigation—namely, that the ratio L/b is not sufficient to determine the strength of such beams. On the contrary, in addition to the L/b-ratio the characteristics of the cross section expressed by the quantities B' and C' (Eqs. 34) are of major importance for determining safe design stresses. Therefore, design specifications probably

NOTE.—This paper by George Winter was published in June, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1943, by T. McLean Jasper, and C. M. Goodrich; December, 1943, by J. P. Wadleck; and February, 1944, by H. D. Hussey.

[8] Asst. Prof., Civ. Eng., Cornell Univ., Ithaca, N. Y.

[8a] Received by the Secretary March 15, 1944.

should not be limited to one single curve for I-beams of any shape, but a set of curves or formulas should be established to take account not only of the L/b-ratio but also of the characteristics of the cross section. This becomes even more important if the stipulations are also to apply correctly to built-up girders on the one hand and to junior I-beams on the other as observed by Mr. Wadleck. If members of this type are considered, the variations in strength for a given L/b-ratio will be found to be still larger than those for the standard I-beams in Figs. 4 and 7. These practical design formulas could well be of the Rankine type, as proposed by Mr. Hussey. A still better approximation to the secant curve can be obtained by taking a formula of the Johnson parabolic type for the lower range of L/b combined with one of the Rankine type for the higher range.

Mr. Goodrich's contention that "the author's mathematics yield no rule for the designer" is best answered by Mr. Hussey's very practical conclusions. The writer never intended to propose definite design specifications in his paper since the establishment of such codes is the task of the appropriate engineering organizations rather than of any single individual. When Eüler and others developed a rational theory of column behavior they did not, at the same time, attempt to prescribe definite design requirements; yet the total knowledge of the design of compression members would be poor indeed were it not for the fundamental investigations of these men.

Mr. Goodrich cites Professor Moore's tests in 1913 and attempts to compare the results with Fig. 4 of the paper. Such a comparison is not quite appropriate for the following reasons:

(*a*) Fig. 4 is drawn for one definite yield point only, 36,000 lb per sq in. The writer could not find any reference by Professor Moore (21)[8b] to the yield points of this particular series of tests, but some other yield points reported in the course of the same investigation were as high as 40,000 lb per sq in. and more. Thus, the curves in the writer's paper, drawn for a yield-point stress of 36,000 lb per sq in., cannot coincide with Professor Moore's value of s = 37,500 to 42,500 lb per sq in. for $L/i = 0$. A further reason for this discrepancy is the well-known fact that short steel beams, with L/i close to zero, fail at loads higher than those which cause yielding in the outermost fiber (8). Since, however, incipient yielding is generally regarded as the limit of structural usefulness, this state rather than that of complete failure is analyzed by the secant formula. The difference between this limiting load and that causing complete failure decreases rapidly with increasing slenderness.

(*b*) Fig. 4 is drawn for an entirely arbitrary degree of imperfection $k = 0.25$. A comparison of Professor Moore's results with those for $k = 0.10$ would have yielded entirely different results. Probably, for the accuracy of a careful experimental investigation $k = 0.25$ is much too high.

(*c*) From a study of Professor Moore's testing arrangement complete freedom of twist and of lateral deflection does not seem to have been achieved. A rather minute restraint of lateral freedom at or near the mid-length of a column will increase the buckling strength considerably. The same is true for

[8b] Numerals in parentheses, thus: (21), refer to corresponding items in the Bibliography (see Appendix I of the paper), and at the end of discussion in this issue.

buckling of beams. Professor Moore's values, therefore, are likely to be rather high as compared with those that would be obtained in tests in which complete freedom of movement was provided. Such freedom in beam tests in standard testing machines is extremely difficult to achieve. To the writer's knowledge, the only satisfactory, although cumbersome, way of doing so is to suspend actual dead loads from the test beams rather than to apply the loads by means of a testing machine.

In connection with Professor Moore's tests it is interesting to cite corresponding tests on junior I-beams by Messrs. Ketchum and Draffin (15), quoted by Mr. Jasper. The formula given by these investigators,

$$s = 24{,}000 - 40\left(\frac{m\,L}{i}\right) \qquad (82)$$

gives values which are very considerably below those of Professor Moore's original formula, Eq. 71, despite the practical identity of testing arrangements. This divergence once again emphasizes the fact that neither the flange width b nor the radius of gyration i is sufficient to describe the lateral strength of beams. Empirical column formulas expressed in terms of L/i give reasonable results for the sole reason that L/i is the only shape factor on which the strength of columns depends. This is shown by all rigorous analytical investigations of column behavior. Conversely, all rigorous analytical investigations of buckling of beams (1)(5)(6)(10)(12), including the present one, show most decidedly that neither L/b nor L/i is sufficient to determine the strength of slender beams. Thus, none of the approaches proposed by Mr. Goodrich, based on L/i, will give satisfactorily accurate results. This has been recognized, for instance, in recommended design procedures for aluminum alloy products (24), in which the allowable design stress is determined by use of an "equivalent radius of gyration" whose value depends on precisely the same quantities as the writer's B' and C' and is in no way identical with Mr. Goodrich's ordinary radius of gyration i.

Mr. Goodrich's discussion of the value of the torsional constant K is somewhat extraneous to the subject of this paper. The writer gave an approximate formula, Eq. 50, and references to the most recent and complete information in this connection. Since the validity of this information is generally recognized, there seemed to be no need for a detailed discussion of this constant except as far as its value for built-up sections is concerned.

Mr. Wadleck's mathematical extension of the paper represents an interesting generalization of the writer's results. Eq. 73*b*, expressing s in terms of the ratio of the applied to the critical moment, M/M_c, agrees with general concepts on stability, in particular with the Southwell method (5*f*) of determining deflections of imperfect compression members.

Like Mr. Wadleck, the writer did attempt to express the secant formula in terms of a simpler, rational function, but met with the same difficulties as did Mr. Wadleck. However, the practical inconvenience of solving a secant formula can be overcome by approximating it with a combination of a Johnson and a Rankine type formula, as noted herein. A set of suitably chosen curves

of this nature would eliminate any necessity for the designer to solve the secant formula directly. Graphs consisting of a large number of curves are widely used in reinforced concrete design where rather complicated equations are involved, like those governing the design of eccentrically loaded columns. The writer cannot see any objection to the introduction of graphs of this kind in steel design if by so doing design procedures are improved and economy is achieved.

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(5) **"Theory of Elastic Stability,"** by S. Timoshenko, McGraw-Hill Book Co., Inc., 1938. (*a*) p. 240. (*b*) p. 88, Eq. 66. (*c*) p. 13, Eq. 25. (*d*) p. 98. (*e*) p. 499. (*f*) p. 177.

(6) **"Action of Deep Beams Under Combined Vertical, Lateral and Torsional Loads,"** by C. O. Dohrenwend, *Journal of Applied Mechanics*, Vol. 8, No. 3, 1941, p. A-130.

(8) "Discussion by George Winter of **"Theory of Limit Design,"** by J. A. Van den Broek, *Transactions*, Am. Soc. C. E., Vol. 105 (1940), p. 673.

(10) **"Lateral Stability of Unsymmetrical I-Beams and Trusses in Bending,"** by George Winter, *Proceedings*, Am. Soc. C. E., December, 1941, p. 1851. (*a*) p. 1855, Eq. 14*b*. (*b*) p. 1856, Eq. 22*b*. (*c*) p. 1856, Eq. 22*a*.

(12) Discussion by Robert K. Schrader of **"Lateral Stability of Unsymmetrical I-Beams and Trusses in Bending,"** by George Winter, *ibid.*, February, 1942, p. 299. (*a*) Eqs. 55 and 62.

(15) *Bulletin No. 241*, Eng. Experiment Station, Univ. of Illinois, Urbana, 1932.

(21) **"The Strength of I-Beams in Flexure,"** *Bulletin No. 68*, Univ. of Illinois, Urbana, 1913.

(24) **Structural Aluminum Handbook,** Aluminum Co. of America, 1938, p. 48.

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DISCUSSIONS

ANALYSIS OF RIGID FRAMES BY SUPERPOSITION

Discussion

BY F. S. MERRITT, RALPH W. STEWART, AND JOHN E. GOLDBERG

F. S. MERRITT,[4] JUN. AM. SOC. C. E.[4a]—Although no new method of analysis is proposed (see "Synopsis"), this paper is of value in illustrating the application of the principle of superposition to different types of frames. As Professor Wilson acknowledges, the principle of superposition is usually stated in textbooks on structural analysis; but, contrary to his opinion, it is also strongly emphasized in texts discussing moment distribution, since it is the most convenient tool for analyzing rigid frames by this method. However, nowhere in this paper is the principle of superposition, the subject of the paper, clearly stated or defined; nor are the limits of its validity described.

In the outline of the method of procedure, five steps are given for the solution of any rigid frame problem. Hardy Cross and N. D. Morgan, Members, Am. Soc. C. E., describe this same procedure simply and succintly as follows (3*a*):[4b]

> "In analyzing a bent by moment distribution we first assume no movement of the joints and analyze for this condition, find by statics the force necessary to prevent such movements and find the moments which it would produce, and then take the difference of the two results."

The writer has developed the following procedure (16):

> "1. Apply forces to the structure to prevent sidesway while the fixed-end moments due to loads are distributed.
> "2. Compute the moments due to these forces.
> "3. Combine the moments obtained in Steps 1 and 2 to eliminate the effect of the forces."

When moment distribution by the method of moment ratios (16) is used, there is never any need to revert to the slope-deflection method, because the ques-

NOTE.—This paper by David M. Wilson was published in February, 1944, *Proceedings*.

[4] Field Service Engr., D. W. Haering & Co., New York, N. Y.

[4a] Received by the Secretary February 21, 1944.

[4b] Numerals in parentheses, thus: (3*a*), refer to corresponding items in the Bibliography (see Appendix of the paper), and at the end of discussion in this issue.

tion of convergence does not arise. The author's recommendation in conclusion 4, therefore, is not generally valid.

In applying either of these last two procedures, sketches may be desirable, but they are not necessary. From a practical viewpoint, the preparation of sketches requires time, and, where the calculations can follow a rigid procedure in which mistakes can be avoided without the use of sketches, such a procedure is recommended. In the classroom, sketches are useful in the introduction of an unfamiliar subject, but, in a drafting room, where time is money and a designer is assumed to be familiar with structural analysis, time-saving procedures should be adopted. Consequently, step 1 of the paper should be optional with the designer and should be included as a suggestion to facilitate determination of signs rather than as a step in the procedure.

In analyzing rigid frames, a designer generally wishes to obtain separately the effects of dead load, live load, wind, etc. The determination of live load moments might require the plotting of influence lines. In reality, this is the rigid frame problem, as contrasted to the classroom problem of a given structure with a fixed loading condition. Professor Wilson does not state how his procedure could be used under changing load conditions without the necessity of going through all five steps for each change in position or magnitude of loads. As a matter of fact, the writer has used the three steps outlined herein as the base of a simple procedure in which the principle of superposition is utilized to obtain the separate effects of varied loading conditions without the necessity of distributing fixed-end moments for every condition (16). The practice of computing and placing final moments on sketches, as recommended in steps 2 and 3 by the author, is cumbersome in view of the great number of sketches required and the difficulty in making summations and combinations from them.

The writer agrees with all the conclusions of the paper except conclusion 4.

RALPH W. STEWART,[5] M. AM. SOC. C. E.[5a]—In his opening sentence Professor Wilson states that the purpose of his paper "is to demonstrate the usefulness of the principle of superposition in the analysis of rigid frames by the slope-deflection and moment distribution methods." From an examination of the demonstrations the writer concludes that, for multistory frames with sloping columns and for the Vierendeel truss, the author has accomplished his purpose.

However, in his treatment of one-story and two-story frames with vertical columns, Professor Wilson introduces undesirable complexities and also so much unnecessary arithmetic that the writer does not consider this part of the paper useful. To uphold this conclusion the writer presents Fig. 11, which is the bent shown in Fig. 1, using the moment values in Eqs. 4. By equating the sums of the moments about joints B and C, respectively, to zero and then consolidating terms the author obtains Eqs. 5, containing three unknown quantities. To solve them he introduces Eqs. 7 and 8 from which $E R$ may be evaluated; and then Eqs. 5 can be solved for θ_B and θ_C.

[5] Bridge Engr., City of Los Angeles, Los Angeles, Calif.

[5a] Received by the Secretary March 20, 1944.

Fig. 12 is a graph of the traverse of the elastic curves of Fig. 11. In Fig. 12 the geometrical values of the angles of flexure in terms of θ_B and θ_C are shown. For the beam each angle value is the same as the parenthetical term in a slope-deflection equation for this member. The properties of these angles have been published (17). They are located at the one-third points of the members.

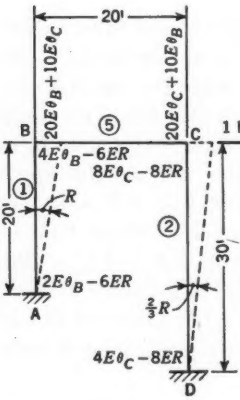

FIG. 11

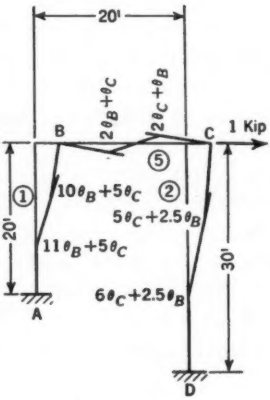

FIG. 12

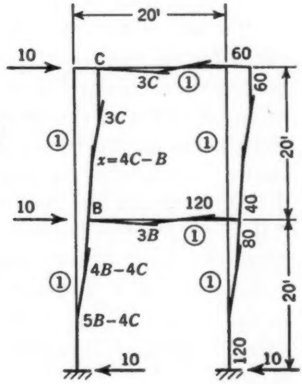

FIG. 13

Since the left column is only one fifth as stiff as the beam, its angle of flexure for the moment at B will be five times as great as the flexure angle in the beam, or $10\,\theta_B + 5\,\theta_C$. To obtain the flexure angle for M_A it is only necessary (from the geometry of the figure) to add θ_B to the flexure angle for the moment at B, giving $11\,\theta_B + 5\,\theta_C$. A similar procedure gives the flexure angles for the right column. The unknown R does not appear in these angle values. A property of an elastic curve traverse is that a flexure angle multiplied by the member stiffness equals the moment that the angle measures. By multiplying the flexure angles by the K-values a set of relative moment values for Fig. 12 is obtained. The geometry of the flexure of the columns gives the deflections at points B and C. Writing the equilibrium equation, in which the sum of the column shears equals 1 kip, and the geometrical equation, which expresses the fact that the deflections at B and C are equal, and consolidating terms:

$$1.383\,\theta_B + 1.233\,\theta_C = 1 \qquad (33a)$$

and

$$5.5\,\theta_B = 3.667\,\theta_C \qquad (33b)$$

These equations, with only two unknown quantities, replace Eqs. 5 with three unknown quantities.

Solving, $\theta_B = 0.3093$ and $\theta_C = 0.464$. The flexure angles can now be evaluated. Multiplying these angles by the K-values of the members gives a set of moments which check Professor Wilson's moments to slide-rule accuracy.

To solve the two-story symmetrical frame illustrated by Fig. 4, the author's procedure involved the following work: Steps 1 and 2, two end-moment distribution procedures for the frame; and, step 3, the development and solution of two simultaneous formulas (Eqs. 13). The two end-moment distribution operations were unnecessary as all of the work accomplished by these three stages of the author's solution can be accomplished by two simultaneous equations taken from the traverse diagram in Fig. 13.

To reduce the labor of typing and engraving, B and C have been substituted for θ_B and θ_C in Fig. 13 and the equations relating to this frame. The beam flexure angles $3\,B$ and $3\,C$ are shown in Fig. 13. Since the members are of equal stiffness, the uppermost column flexure angle is also $3\,C$. By angle summation for the upper column: $C + 3\,C - x - B = 0$, or $x = 4\,C - B$, as shown. Balancing moments about point B gives $4\,B - 4\,C$ as the upper flexure angle for the lower column. By angle summation $B + 4\,B - 4\,C$, or $5\,B - 4\,C$, is the bottom flexure angle as shown. Top-story shear balance gives: $7\,C - B = 100$; bottom-story shear balance gives: $9\,B - 8\,C = 200$; and solving: $B = 40$ and $C = 20$. The moments (which are the same as those in the paper) appear in the right half of Fig. 13. The labor involved in this solution is much less than that required by the author's multistage demonstration.

Even if the principle of superposition were mandatory for all the demonstration problems, Professor Wilson has, in general, performed more than double the necessary preliminary work by restricting his paper to slope deflection and end-moment distribution. For example, the work in Fig. 5(*a*) involved one hundred and twenty-eight arithmetical operations. With the traverse method, the results can be duplicated with thirty-seven arithmetical operations and two slide-rule settings from one of which seven readings are taken; and from the other setting, four readings.

The traverse solution also has some automatic checks which end-moment distribution does not. For example, in Fig. 5(*a*), M_{CE} should be exactly six times M_{EC}. The author's solution yields $6 \times 208 = 1{,}268$. The correct value of M_{EC} is 211.3. This difference in moments is not important but the fact that the traverse method offers checks not offered by end-moment distribution is important. Also the author's solution is applicable to only one position of the live load whereas the traverse solution can be used for any position of the live load.

For Fig. 4(*c*) all the arithmetical work needed to compute the moments by the traverse method is as follows: $2 + 3 = 5$; $4\,x = 6$; $x = 1\frac{1}{2}$; $3\,x = 4.5$; $3 + 10 - 4.5 = 8.5$; and $6 \times 1 = 6$. Finally:

$$\frac{6\,E\,R}{8.5} \times \begin{cases} 4.5 = 3.176\,E\,R \\ 5.0 = 3.529\,E\,R \\ 3.0 = 2.121\,E\,R \\ 2 = 1.412\,E\,R \\ 1 = 0.706\,E\,R \end{cases} \qquad \begin{matrix} 3.17\,E\,R \\ 3.54\,E\,R \\ 2.12\,E\,R \\ 1.42\,E\,R \\ 0.72\,E\,R \end{matrix}$$

(Left column: Traverse results; right column: Wilson results)

The detail of Professor Wilson's end-moment distribution is not given, but it must certainly have involved several times as much work as the foregoing. The traverse solution of Fig. 4(*b*) is equally simple.

To summarize: The paper is of value to engineers who design framed structures. Its usefulness is considerably impaired, however, by unnecessarily complex solutions for some of the problems due to the exclusion of the elastic curve traverse from the scope of the work.

JOHN E. GOLDBERG,[6] ASSOC. M. AM. SOC. C. E.[6a]—A commendable résumé of the application of the principle of superposition to the analysis of certain types of rigid frame structures is presented by Professor Wilson. The paper is an excellent coordination of this principle with the particular types of structures described.

The method of superposition will give a reasonable analysis in many cases where the solution by more direct methods may be comparatively tedious. This is true because in direct methods of analysis a large number of simultaneous equations must be solved, whereas in the method of superposition a limited number of modes of deformation are first analyzed and subsequently used as parameters in the solution of the complete framework. Obviously, using the latter procedure there are a smaller number of equations to solve simultaneously. The method of superposition is particularly applicable to frames of what may be termed intermediate complexity.

The writer has used similar methods for a number of years in conjunction with slope deflection and moment distribution for the analysis of several classes of structures—particularly Vierendeel trusses with cambered chords, A-frames, and simple or gabled bents—and also for the analysis of sidesway effects induced by unsymmetrical vertical loading or by transverse loading of building frames not more than a few stories in height. In such problems the method of superposition provides a simple and practicable method of analysis, particularly when used in conjunction with moment distribution or slope deflection (18)(19) and successive approximations or successive corrections. Certain of these problems have been analyzed by various writers in a number of texts and papers. The gabled bent, for example, has been "popular" in technical publications. The analysis of simple bents and bents with inclined legs likewise has been presented in several texts.

Some problems discussed by Professor Wilson may be solved at least as easily by direct methods without recourse to the method of superposition. Certain simple bents are generally in this class. A structure that may be analyzed easily by more direct methods is the Vierendeel truss with equal parallel chords or the structurally similar two-column bent with parallel equal columns. The attention of the writer was directed to this problem a number of years ago by the comparative impracticability of applying the usual popular methods to such structures when the outside members (that is, chords in the Vierendeel trusses or columns in the bents) are relatively stiff, say, five

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[6] Structures Engr., Consolidated Vultee Aircraft Corp., San Diego, Calif.

[6a] Received by the Secretary March 30, 1944.

or more times as stiff as the interior members. In such cases moment or shear distribution is extremely slow in converging to the exact solution.

For a simple and practicable solution of this problem the writer derived the following exact formula from the basic slope-deflection equations:

$$\theta_n = \frac{1}{6\,K_{Gn} + K_{Cn} + K_{Co}}\left(\frac{M_n + M_o}{2} + K_{Cn}\,\theta_m + K_{Co}\,\theta_o\right) \ldots\ldots (34)$$

in which K_{Gn} and K_{Cn} are the relative stiffnesses of the web members and of the chord members, respectively; and M_n is the product of the transverse external shear acting on the nth panel times the longitudinal dimension of the panel—that is, the distance between interior members. When the known K and M values are substituted, Eq. 34 reduces to an expression of the simple form,

$$\theta_n = A\,\theta_m + B\,\theta_o + C \ldots\ldots (35)$$

A similar equation is set up for each panel point along either chord. The series of equations thus obtained converge very rapidly. For example, in cases where the chords are from eight to ten times as stiff as the interior members, three or four cycles of these extremely simple calculations will give an accuracy better than twenty cycles (approximately) by the usual moment distribution or shear distribution procedure. The web-member end moments are given by the formula,

$$M_{Gn} = -\,3\,(K_{Gn}\,\theta_n) \ldots\ldots (36a)$$

and the chord-member end moments by the formula,

$$M_{Cnm} = \frac{M_n}{4} + \left(\frac{\theta_m - \theta_n}{2}\right) K_{Cn} \ldots\ldots (36b)$$

Obviously, a number of problems not treated by Professor Wilson may also be analyzed by the method of superposition. The method of superposition is an extremely general and practicable method, and Professor Wilson is to be commended for his clear and concise outline.

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(18) **"Simplified Methods for the Analysis of Multiple Joint Rigid Frames,"** by George A. Maney and John E. Goldberg, *Bulletin*, Northwestern Univ., Evanston, Ill., Vol. 33, No. 7, October 17, 1932.

(19) **"Vertical Load Analysis of Rigid Building Frames Made Practicable,"** by John E. Goldberg, *Engineering News-Record*, November 12, 1931, pp. 770–772.

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DISCUSSIONS

SHEAR AND BOND STRESSES IN REINFORCED CONCRETE

Discussion

By F. R. Shanley, B. J. Aleck, Dean Peabody Jr., and William E. Wilbur

F. R. Shanley,[7] Esq.[7a]—Airplane structural designers frequently abandon the classical formula for transverse shear stress and use a more general method[8] in which the shear flow is determined directly from the differences in axial force at adjacent cross sections along the beam. In effect, this amounts to a direct use of Eq. 2. If the axial stresses have been determined at relatively close stations, the additional work involved to obtain the shear and bond stresses is negligible. The use of the method for bond stresses has been demonstrated by Fred B. Seely.[9]

The main reason for adopting this method in airplane analysis was that the conventional shear stress formula does not account for variations in the cross section along the beam. For instance, a taper in depth may relieve the shear stresses greatly, but this cannot be shown by a formula that deals with only one section at a time. The general method would seem particularly useful for beams in which some of the steel bars are placed at an angle. Another advantage is that the effects of all variables, including compression loading, are included in computing the bending stresses and need not be considered again in determining shear or bond stresses.

The "unit" method, as it is called, is quite sensitive to abrupt changes in cross section. If the sections used for analysis are fairly close together, the resulting shear stresses will appear abnormally high. Some of this effect is relieved by local distortion of the cross section, or by yielding of a ductile material, but the designer regards abnormally high shear stresses as danger signals, indicating poor structural continuity. This might be rather important in dealing with a brittle material like concrete.

Note.—This paper by Stanley U. Benscoter and Samuel T. Logan was published in March, 1944, *Proceedings*.

[7] Div. Engr., Structural Research, Lockheed Aircraft Corp., Burbank, Calif.

[7a] Received by the Secretary March 31, 1944.

[8] "Unit Method of Beam Analysis," by F. R. Shanley and F. P. Cozzone, *Journal of the Aeronautical Sciences*, Vol. 8, No. 6, April, 1941, p. 246.

[9] "Resistance of Materials," by Fred B. Seely, John Wiley & Sons, Inc., New York, N. Y., 2d Ed., 1935, p. 386.

B. J. ALECK,[10] JUN. AM. SOC. C. E.[10a]—As the authors indicate, common formulas sometimes yield conservative shear and bond stresses, when a member is subjected to axial load. The writer finds the authors' results somewhat incomplete, however, and not always conservative. For example, using the authors' variables:

$$T = n\,A_t\left(\frac{M}{S_t} - \frac{P}{A}\right) \qquad (16)$$

in which A is the area of the transformed cross section; and the unit shear stress (with P constant) is:

$$v = \frac{1}{b}\frac{dT}{dx} = \frac{n\,A_t}{b}\left(\frac{\frac{dM}{dx}}{S_t} - \frac{M}{S^2{}_t}\frac{dS_t}{dx} + \frac{P}{A^2}\frac{dA}{dx}\right) \qquad (17)$$

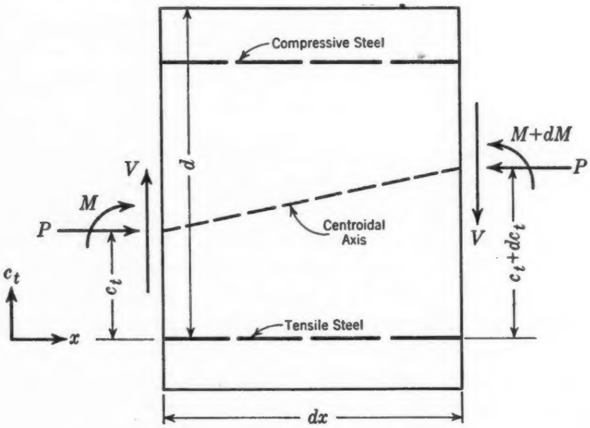

FIG. 7.

Considering an element as shown in Fig. 7:

$$\frac{dM}{dx} = V - P\,\frac{dc_t}{dx} \qquad (18)$$

Hence

$$v = \frac{n\,A_t}{b}\left(\frac{V}{S_t} - \frac{P}{S_t}\frac{dc_t}{dx} - \frac{M}{S^2{}_t}\frac{dS_t}{dx} + \frac{P}{A^2}\frac{dA}{dx}\right) \qquad (19)$$

The authors' solution evidently corresponds to the result if the last three terms in Eq. 19 were to vanish (see Eq. 8*a*). As the prospect of evaluating the derivatives in Eq. 19 is discouraging, the same formula will be derived in terms of simpler variables; thus, if M_s is the moment of all external forces about the tension steel and $j\,d$ is the distance between the resultant compressive force in the concrete and compression steel and the force in the tension steel:

$$T = \frac{M_s}{j\,d} - P \qquad (20)$$

[10] Research Engr. (Materials and Structures), Curtiss-Wright, Buffalo, N. Y.

[10a] Received by the Secretary April 3, 1944.

and

$$v = \frac{1}{b}\frac{dT}{dx} = \frac{\dfrac{dM_s}{dx}}{b\,j\,d} - \frac{M_s}{j^2\,b\,d}\frac{dj}{dx} = \frac{V}{b\,j\,d} - \frac{M_s}{j^2\,b\,d}\frac{dj}{dx} \qquad (21)$$

The resultant resisting compressive force, C, is equal to

$$C = b\,d\,f_c\left[\frac{k}{2} + \left(1 - \frac{\alpha}{k}\right)(n-1)\,p_c\right] \qquad (22a)$$

Also,

$$P\,e = M_s = b\,d^2 f_c\left[\frac{k}{2}\left(1 - \frac{k}{3}\right) + (1-\alpha)\left(1 - \frac{\alpha}{k}\right)(n-1)\,p_c\right] \qquad (22b)$$

and

$$P = C - T = b\,d\,f_c \times \left[\frac{k}{2} + \left(1 - \frac{\alpha}{k}\right)(n-1)\,p_c - \left(\frac{1}{k} - 1\right) n\,p\right] = \text{constant} \qquad (22c)$$

Eqs. 22 are well known. Moving from one section to the next, k and f_c change with x. Differentiating Eq. 22c with respect to k:

$$\frac{df_c}{dk} = -\frac{f^2_c\,b\,d}{P}\left[\frac{1}{2} + \frac{\alpha}{k^2}(n-1)\,p_c + \frac{1}{k^2}\,n\,p\right] \qquad (23)$$

Differentiating Eq. 22b with respect to x, using Eq. 23:

$$\frac{dk}{dx} = \frac{V}{f_c\,b\,d^2} \times \frac{1}{\dfrac{1}{2} - \dfrac{k}{3} + \dfrac{\alpha}{k^2}(1-\alpha)(n-1)\,p_c - \dfrac{e}{d}\left[\dfrac{1}{2} + \dfrac{\alpha}{k^2}(n-1)\,p_c + \dfrac{1}{k^2}\,n\,p\right]} \qquad (24)$$

$$j = \frac{M_s}{C\,d} = \frac{\dfrac{k}{2}\left(1 - \dfrac{k}{3}\right) + (1-\alpha)\left(1 - \dfrac{\alpha}{k}\right)(n-1)\,p_c}{\dfrac{k}{2} + \left(1 - \dfrac{\alpha}{k}\right)(n-1)\,p_c} \qquad (25)$$

$$\frac{dj}{dk} = \frac{b\,d\,f_c}{C}\left\{\frac{1}{2} - \frac{k}{3} + \frac{\alpha}{k^2}(1-\alpha)(n-1)\,p_c - j\left[\frac{1}{2} + \frac{\alpha}{k^2}(n-1)\,p_c\right]\right\} \qquad (26)$$

and

$$\frac{dj}{dx} = \frac{dj}{dk}\frac{dk}{dx} = \frac{V}{C\,d}B \qquad (27)$$

In Eq. 27,

$$B = \frac{\dfrac{1}{2} - \dfrac{k}{3} + \dfrac{\alpha}{k^2}(1-\alpha)(n-1)\,p_c - j\left[\dfrac{1}{2} + \dfrac{\alpha}{k^2}(n-1)\,p_c\right]}{\dfrac{1}{2} - \dfrac{k}{3} + \dfrac{\alpha}{k^2}(1-\alpha)(n-1)\,p_c - \dfrac{e}{d}\left[\dfrac{1}{2} + \dfrac{\alpha}{k^2}(n-1)\,p_c + \dfrac{n\,p}{k^2}\right]} \qquad (28)$$

Therefore,

$$v = \frac{V}{b\,j\,d} - \frac{M_s}{b\,j^2\,d}\frac{dj}{dx} = \frac{V}{b\,j\,d}(1 - B) \quad \text{.............. (29)}$$

Eq. 29 represents the complete answer. To facilitate simple comparisons it is convenient to compare results when $p_c = 0$. In this case,

$$v = \frac{V}{b\,j\,d}\left[\frac{2\,p\,n\,(1 + k)}{k\,(k + 4\,p\,n)}\right]_{p_c=0} \quad \text{.................. (30)}$$

whereas the authors' formula leads to

$$v = \frac{V}{b\,j\,d}\left[\frac{\left(1 - \frac{k}{2}\right)\left(1 - \frac{k}{3}\right)}{1 - k + \frac{k^2}{3} + \frac{k^3}{12\,p\,n}}\right]_{p_c=0} \quad \text{.............. (31)}$$

Eq. 30 is readily plotted on a graph as $\frac{v\,b\,d}{V}$ versus $p\,n$, for k held constant at 1.0, 0.9, 0.8, etc. Some idea of possible divergence of answers is afforded when $k = 0.6$ and $p\,n = 0.05$. Then, Eq. 30 yields 52.4% of Eq. 31 and 33.3% of the code value.

More important is the lack of conservatism of both the code and authors' formulas when the axial load is tensile. Thus if $k = 0.20$ and $p\,n = 0.12$, the shearing stress according to Eq. 30 is 212% of that given by the code and 206% of that given by Eq. 31.

After this theoretical discussion, the inadequacy of these results should be emphasized. Some questions that deserve consideration are:

1. How good a criterion is shear for diagonal tension?[11] One authority states: "The vertical shearing stress is not the numerical equivalent of the diagonal tension stress; nor is there any definite relation between them."

2. How accurate is the assumed stress distribution?

   (*a*) Is the concrete carrying tension (in flexure)? How much?
   (*b*) Is the stress diagram linear over the transformed section?
   (*c*) How much load does the compressive steel carry?[12]

3. Is the allowable shear stress given by code to be used with the new formula? Tests should be conducted, especially when axial load is tension.

4. What is the correct value of p to use in the region where bars are lapped? What allowance should be made for variation of effective p-value in this region?

DEAN PEABODY, JR.,[13] M. AM. SOC. C. E.[13a]—An alternate method of computing the bond stresses u in tensile reinforcing steel, and the shear stresses v in the concrete between the tensile steel and the neutral axis, is developed in this

[11] "Design of Concrete Structures," by L. C. Urquhart and C. E. O'Rourke, 4th Ed., McGraw-Hill Book Co., Inc., New York, N. Y., 1940, p. 93.

[12] "Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete," *Proceedings*, Am. Soc. C. E., June, 1940, p. 45, Section 804(c).

[13] Associate Prof., Bldg. Constr., Mass. Inst. of Tech., Cambridge, Mass.

[13a] Received by the Secretary April 4, 1944.

paper. Also, the variation of shear stress v between the neutral axis and the extreme fiber in compression is discussed. For these computations the transformed area procedure is used. The authors consider only the case of sections that have cracked close enough to the neutral axis for the tensile stresses in the concrete to be neglected.

Beam Without Thrust.—Eqs. 8 use the section modulus S_t, which can be computed by Eq. 12*b* or 13. To compute S_t the neutral axis ratio k must be determined by the usual reinforced concrete equations, by tables or plots, or by the use of the transformed area. A designer working "alternatively with steel, timber, and concrete" could determine j at once, after reviewing the theory sufficiently to compute k. Only for the beam with tension steel does $j = 1 - \frac{k}{3}$, as stated in the "Notation" and, also, immediately following Eqs. 15. However, if compression steel is used, the moment arm $j\,d$ is the distance between the resultant tensile force and the resultant of the compressive forces, and the moment arm ratio j no longer equals $1 - \frac{k}{3}$. If j is determined correctly for beams with compression steel, Eqs. 15 are not "somewhat in error" but give the same result as Eqs. 8. The use of Eqs. 15 or 8 would seem a matter of personal opinion.

Member with Thrust.—This material is valuable, as the designer has not often investigated the shear stresses in columns subjected to bending, nor included the direct thrust in the design of beams. The authors discuss only the case of compressive stresses on part of the section with the concrete cracked on the tension side. This is a common case for a beam with a direct thrust, but many columns have sections with compressive stresses over the entire area. In such a case the easiest procedure would employ the transformed area, determine the centroidal axis, and use Eq. 3. In fact, the writer would rather use Eq. 3 for all cases of thrust: First, because the maximum shear stress occurs at the centroidal axis (as in Fig. 3) and can be determined by this method; and, second, because Eqs. 8 cannot be used for the shear stress near the tension steel until the transformed area computations furnish the centroidal axis, neutral axis, and moment of inertia. Having proceeded thus far with the transformed area, Eq. 3 can be used as expeditiously as Eq. 8*a*.

In the sixth paragraph from the end of the paper, the authors note that Eq. 15*a* does not give the correct shear stress v in the concrete between the tension steel and the neutral axis (Fig. 3). Their estimate of an error of 65% is not correct, because it is assumed that $j = 1 - \frac{k}{3} = \frac{7}{8}$. The distance between the resultant compressive force and the resultant tension force, if denoted $j\,d$, is less than this and the error is greater (the writer computes $j = 0.83$). However, Eq. 15*a* should never be used for members with thrust. It was derived for the case in which the compressive force C equals the tensile force T; and this is never true for members with thrust. In other words, Eq. 15*a* holds when the centroidal and neutral axes coincide.

Diagonal Tension.—Under the heading, "Members with Thrust," the authors state that the shear stress v is equal in magnitude to the diagonal tensile

stress. This is not accurate, because the principal tensile stress s_t on any particle is equal to:

$$s_t = \frac{f}{2} \pm \frac{1}{2}\sqrt{f^2 + 4\,v^2} \quad \text{(32)}$$

in which f = fiber stress. To avoid the use of Eq. 32, the shear stress v has been employed as a measure of diagonal tension.[14] The tensile stress s_t may be 10% to 20% greater than the shear stress v at sections where diagonal tension is important. It has been the costom to specify allowable shear stresses sufficiently low so that the allowable tensile stress s_t will not be exceeded. The tables of allowable shear stresses in the past and present Joint Committee Codes and the Regulations of the American Concrete Institute have never clearly stated this fact.

For a beam of single span supported at the ends, the principal tensile stress s_t will make the greatest angle (45°) with the longitudinal axis of the beam at or near the supports. The particles in these sections have small fiber stresses f and comparatively large shear stresses v. One may assume that the concrete on the tension side has not cracked all the way to the neutral axis, and a tensile stress may be assigned to the concrete whose shear equivalent is $0.02\,f'_c$ or $0.03\,f'_c$, even for particles just above the tension steel. It would seem logical, then, in the authors' method, to include some of the tension area of the concrete in the transformed area and in the computation of the centroidal axis, the moment of inertia, and the shear stress v.

For continuous beams, the maximum negative moment and the maximum shear force both occur at the support. The assumption of a section cracked to the neutral axis is logical, but designers still assume that the concrete takes a tensile stress whose shear equivalent v equals $0.02\,f'_c$ or $0.03\,f'_c$ when computing diagonal tension. In continuous beams, the greatest inclination of the principal stress s_t, near the bottom of the beam, will occur close to the point of inflection of the moment diagram. At this point, about one fifth to one fourth the span length from the support, the fiber stresses f are small and the entire concrete area down to the tensile steel might well be used to compute the transformed area and the shear stress. Engineers have never chosen to take time enough to attain this accuracy. However, for members with thrust, cases will arise in which the tension in the concrete is within its allowable stress, and the authors may well consider sections where a part, or the whole, of the concrete tension area is included in the transformed area.

The authors do write briefly about sections whose particles are wholly in compression in the third paragraph from the end of the paper. The statement, "There is no diagonal tension in this region," for such particles is not completely accurate. The principal tensile stress s_t equals:

$$s_t = \frac{f_c}{2} - \frac{1}{2}\sqrt{f^2_c + 4\,v^2} \quad \text{(33)}$$

and will have a small, probably negligible, value.

[14] Regulations, A.C.I., 1941, Section 1205a.

The writer has found this paper stimulating but confesses that he leaves it with a greater respect for the transformed area solution for all cases except the beam without thrust.

WILLIAM E. WILBUR,[15] M. AM. SOC. C. E.[15a]—The subject of combined flexure and direct stress in concrete beams has always been a troublesome one for designers to analyze. In a measure, this is due to two assumptions made in concrete design—namely, a straight-line variation of fiber stress, and no tension in the concrete. It is no part of this discussion to question these assumptions, since they lead to simple and sufficiently accurate formulas. They will be assumed as correct.

In the first part of their paper, the authors derive expressions for unit shear and bond stress in beams subject to bending only. In order not to confuse the nomenclature, let $z\,d$ be the depth from the centroid of the tensile fiber stresses to the centroid of the compressive fiber stresses. Then, for any beam, of any shape, the laws of mechanics give the unit shear at the neutral axis as

$$v = \frac{V}{b\,z\,d} \qquad (34a)$$

When b is constant, this is the maximum unit shear. For a rectangular section, such as a timber beam, z has the value of $\frac{2}{3}$, and the expression becomes

$$v = \frac{3\,V}{2\,b\,d} \qquad (34b)$$

For a concrete beam without compressive steel, $z = 1 - k/3$, or "j," and Eq. 34*a* becomes Eq. 15*a*. For an I-beam or plate girder with web thickness b, z is nearly unity, and substitution in Eq. 34*a* gives the common approximation

$$v = \frac{V}{b\,d} \qquad (34c)$$

When there is compressive steel in a concrete beam, the value of z will be exactly the same as j only when the compressive steel is at the centroid of the compressive concrete fiber stresses. Substitution of $k/3$ for α in Eqs. 11 and 12 will be found to result in the same values of Eqs. 15 as those given by the authors.

In actual design, the area of compressive steel is usually relatively small, and the value of α is not far from $k/3$. It follows, then, that the value of v will not vary greatly from the value $\frac{V}{b\,j\,d}$ (Eq. 15*a*) and that expression will nearly always be sufficiently exact for practical purposes.

In the latter part of their paper, the authors discuss the case of flexure and direct stress. They are correct in their statement (see heading, "Member with Thrust") that "Any change in the value of the thrust or moment changes the location of the neutral axis," and hence all the properties of the section. In

[15] Structural Engr., The Fluor Corp., Kansas City, Mo.

[15a] Received by the Secretary April 10, 1944.

discussion of a previous paper[16] the writer demonstrated a simple method for determining the fiber stresses in this case, without recourse to special diagrams or complicated formulas.

The authors are incorrect in their statement that "The integral Q * * * is always computed about the centroidal axis," and this error leads to the irrational results shown in their Fig. 3.

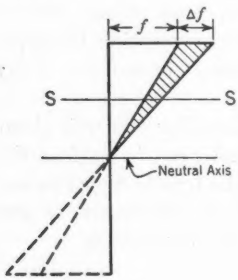

Fig. 8.

Consider a beam subject to flexure, or flexure and direct stress, with maximum fiber stresses at two adjacent sections of f and $(f + \Delta f)$, respectively (see Fig. 8). The hatched area represents the difference between the fiber stresses at the two sections, and is a measure of the horizontal shear. The horizontal shear at S, for example, is measured by the part of the hatched area between S and the extreme fiber. It is obvious that the horizontal shear is maximum at the neutral axis, and cannot be otherwise, on the assumption of no tension in the concrete.

If Eq. 34*a* is applied to the example in Fig. 3, and z is taken as approximately equal to $38 - (\frac{1}{3} \times 19.2)$, the maximum unit shear, between the neutral axis and the tensile steel, is found to be approximately 125 lb per sq in.

The effect of the thrust, then, by decreasing the value of z, is to increase somewhat the maximum unit shear over that in a member with flexure only; and the authors have done well to call this to the attention of designers.

When the thrust is so great as to produce compression over the entire section, the centroidal axis and the neutral axis will coincide, and z will approach the value of $\frac{2}{3}$. The writer cannot agree with the authors' statement (third from the last paragraph) that there is no diagonal tension when the thrust falls within the kern. Diagonal tension will always accompany shear, but in this case the unit diagonal tension will no longer be numerically equal to the unit shear.

16 *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 385.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

GEOLOGY IN HIGHWAY ENGINEERING

Discussion

BY H. E. MARSHALL, K. B. WOODS, AND D. J. BELCHER

H. E. MARSHALL,[12] ESQ.[12a]—The subject of this paper is worthy of the careful attention of both engineers and geologists interested in the efficient operation of a highway system. It presents a comprehensive, up-to-date description of the numerous but often unrecognized problems with which engineers must deal in the design, construction, and maintenance of a highway system in which the application of geological data and methods of study are necessary.

The value of utilizing the services of trained geologists in its highway department has long been realized in Ohio. The department, created in 1904, established the testing laboratory in 1909. The first assignment of the laboratory was to make a geologic study of the sources of aggregate, both developed and undeveloped. The report, prepared by the geologist at that time, has been valuable for the comparison of the wide range of aggregates available in Ohio.

In more recent years studies of the mineral composition of a number of aggregates for service records of portland cement concrete have been made using the petrographic microscope. Although this work has been discontinued at present because of the shortage of trained personnel, it is hoped that it can be continued after the war and that it will indicate a method whereby certain aggregates containing minute quantities of deleterious material, whose effect is not clearly enough recognized by the usual tests, can be identified.

When fourteen dams were constructed for the Muskingum Valley flood control project, approximately 150 miles of highway relocation was necessary. Since much of this relocation involved the construction of fills, a part of which would be inundated, the slopes of these fills had to be protected from erosion by wave action. To find suitable rock for this purpose a considerable amount

NOTE.—This paper by Marshall T. Huntting was published in December, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1944, by W. W. Crosby; March, 1944, by Carl B. Brown; and April, 1944, by F. H. Kellogg, Berlen C. Moneymaker, E. F. Bean, A. T. Bleck, Lyman W. Wood, Philip Keene, and Jacob Feld.

[12] Geologist, Bureau of Tests, Ohio Dept. of Highways, Columbus, Ohio.

[12a] Received by the Secretary March 22, 1944.

of geological surveying was done by highway department geologists in the various flood control basins. In addition, the various stones available (principally sandstone) were studied by the use of the petrographic microscope to determine their composition and probable durability when exposed to the various agencies of weathering on the fill slopes. The location of sources of suitable stone close to the projects has resulted in a considerable saving in the cost of highway work in the district. Similar studies were also made in connection with reconstruction of highway fills damaged by the 1937 flood of the Ohio River.

Geologists have been employed continuously in the study of soils and related problems by the testing laboratory since 1937. Much soil testing for highways consists essentially of an engineering analysis of the physical properties of soil, its strength, and the influence of moisture on its performance. Furthermore, the engineer's training in advanced mechanics and mathematics is a necessity for the testing of the stability of soil by shear and consolidation tests and especially for the application of data obtained from these tests to the solution of problems in the supporting strength of soils for structures and embankments. However, the geologist's working knowledge of the historical processes which have led to the development of soil deposits gives him a picture of their character and distribution which is frequently of more immediate value in dealing with routine questions of soil stability than some of the more elaborate methods of theoretical soil mechanics. For a reasonable and well-rounded study of soils for highway work both the geological approach and the engineering approach must be used.

In the Ohio organization geologists have been found particularly valuable in making field surveys of the soil and rock conditions which will be encountered in the construction of proposed new highways. The mapping of soil profiles, particularly in hilly regions where deep cuts extend into the bedrock, can only be done intelligently if the general geology of the region is understood. Frequently, a detailed study of the geology of the area will be necessary in connection with the soil survey for the project.

In areas where landslides are common, detailed information on the character and structure of the bedrock, in addition to data on the physical properties and thickness of the soil overburden, is necessary to a proper evaluation of the conditions which may cause sliding or under which slides have already developed.

To illustrate the influence of local geology on a specific landslide, the following brief report of a slide which occurred in the southern part of the state is given. A section of the Ohio River Road lying just below a long sweeping curve in the river, where lateral erosion of the channel has kept the river against the valley slope, had been settling and sliding periodically for many years. This section is founded on talus near the toe of the slope. Investigation along the line of the old road showed the depth of poorly consolidated detritus composed of clay and boulders to be so great as to preclude the possibility of satisfactorily holding the road on this line by any practical means. A new road was constructed higher on the valley slope so that most of its length was on a prominent sandstone ledge. However, because of the considerable

excavation necessary to obtain a roadway of sufficient width on the bench, a part of the roadway was constructed of fill extending beyond the old sandstone cliff and resting on the same talus slope occupied by the old road. Several years after construction a slide occurred in this overlapping fill in which about 400 ft of the roadway settled a distance of 5 to 10 ft within a period of two weeks. Investigation of the conditions leading to the development of this slide showed not only that the fill was resting on a great thickness of clay and rock talus but also that water had been flowing through the sandstone stratum down the dip of the rock from a tributary valley which parallels the river at this point about one-half mile away (see Fig. 3). The probability that water

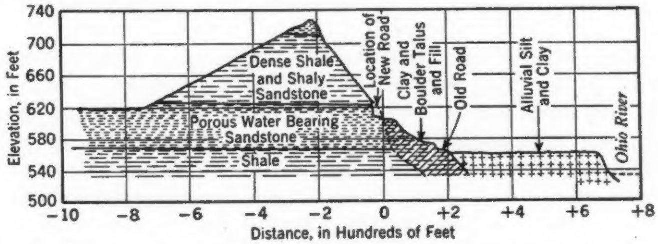

FIG. 3.—WATER SEEPING THROUGH A DIPPING SANDSTONE BED CONTRIBUTED TO THE SATURATION OF THE TALUS SLOPE AND FILL ON WHICH A HIGHWAY WAS CONSTRUCTED IN SOUTHERN OHIO

saturating the fill adjacent to the old rock cliff was coming through this porous sandstone eliminated from consideration any permanent corrective procedures which did not either cut off entirely the water from this source or place the full roadway width on top of the solid rock ledge. The potential water bearing nature of this sandstone was evident only when the inclination of the bedrock as determined from a geological study of the area had been made.

As Mr. Huntting has shown, the geologist by the application of his science can contribute much to the solution of many highway engineering problems, particularly those involving earthwork. However, for engineering purposes, detailed and accurate data must nearly always be obtained by such procedures as test borings and laboratory tests to supplement the more general data obtained from a geological examination. Exact information necessary for the safe construction of an improvement cannot be secured in many cases without detailed investigation. The geologist's services in these problems will be of the greatest value in indicating the need for, and extensiveness of, these studies and in interpreting the results.

K. B. WOODS,[13] M. AM. SOC. C. E.[13a]—The approach used by the author in discussing the relationship of geology to highway engineering by citing typical cases can be extended indefinitely. The periodic recording of these experiences can constitute a valuable addition to engineering literature. The significance of this procedure is emphasized by the fact that highway engineers generally are not familiar with, and lack an appreciation of, the usefulness of geological literature in solving a wide variety of their problems.

[13] Associate Prof., Highway Eng., Asst. Director, Joint Highway Research Project, Purdue Univ., West Lafayette, Ind.

[13a] Received by the Secretary April 14, 1944.

Most states do have, or have had, a geological survey organization which has produced many geological publications containing a vast store of information that can be used to practical advantage by the engineer. Added to these should be other available publications, including in particular the maps and reports produced by the soil surveys of the various states. Pedology and geology go hand in hand and can be useful tools to the highway and airport engineer, particularly in such soil areas as the glacial drift, the great plains, the coastal plains, and in some residual soil areas—including such parent rocks as limestones and shales. In mountainous and semimountainous areas, a knowledge of the rock conditions usually is a primary requirement for satisfactory design and ultimate good performance of the completed engineering structure.

Failure to use available geological information and talents is not entirely the fault of the highway engineer. There is a fundamental difference in the perspectives of geologists and highway engineers. A concrete example is the time element. The geologist thinks in terms of hundreds, thousands, and even millions of years, whereas the highway engineer, in his endeavor to keep pace with the mechanical developments of the automotive industry, is fortunate indeed if he can build and design a highway useful for 50 or even 25 years. The first essential for widespread use of geology in highway engineering is a blending of the two perspectives. The geologist should endeavor to acquaint himself with the broader aspects of engineering and the engineer in turn should decide what phases of geology are applicable to his problems. The engineering and geology schools of the educational institutions in the United States might well be ideally situated to initiate this common understanding.

Referring to the section on "Problems," the following comments will be directed to four of the first six items, namely: (1) Locating suitable road-surfacing material pits and quarries; (2) determining the suitability of various earth materials for surfacing, concrete construction, and other highway uses; (3) subgrade treatment and classification; and (6) preventing and correcting landslides.

In regard to Problem (1), two factors are of major importance—namely, certain land forms and bedrock geology. The geologist well knows that prospecting for gravels and sands in the glacial drift area can be limited to certain land forms including kames, eskers, river terraces, outwash plains, and sand dunes. The engineer is not generally familiar with the geological history of these land forms and as a result his quest for possible sources of granular materials is all too often hit or miss. The geologist, on the other hand, may not be familiar with the engineering requirements of items such as base courses, for instance, and may locate for the engineer's use only those deposits that may yield commercial aggregates. Many states already have available geological maps which show these land forms. The author has listed several of these. A more recent publication than those in the "Bibliography" is the report by H. W. Leavitt and E. H. Perkins (44)[13b] which contains granular materials maps of Maine.

[13b] Numerals in parentheses, thus: (44), refer to corresponding items in the Bibliography (see Appendix of the paper), and at the end of discussion in this issue.

Recent developments in the use of aerial photographs for locating available granular deposits, as well as for interpreting the engineering characteristics of soils, have increased the possibilities of a combined engineering-geological approach to the problem (45). The trained eye can locate granular materials in such photos by noting the drainage patterns, color tone, topography, vegetation, etc., with unfailing accuracy. All deposits of this type are good potential sources of granular materials for highway base courses or even for commercial aggregates in many instances. A knowledge of geology and pedology in this technique is necessary for a correct and accurate interpretation of the evidence included in the photograph.

Another important consideration in the use of information concerning land forms and their effect on highways and airports is that of location. River terraces, for instance, frequently are excellent airport sites because of the granular texture of the soil which provides adequate drainage without the use of many additional man-made appurtenances. Soil survey reports should not be neglected in this respect since detailed maps frequently are available and they can be used to establish grades and to locate highways and airports on the best available materials. Likewise, these same maps, when available, do show the locations of available granular materials.

In regard to Problem (2), the author stresses the fact that research and physical testing of aggregates, used in portland cement concrete construction, have not kept pace with the developments in concrete construction in general. The aggregate has been considered more or less as an inert material, the active ingredient being the cement. As stated by the author, available information disproves this idea. Performance surveys of concrete roads in Kentucky (46) indicate that certain river gravels produced pavements much inferior in quality to those made with other aggregates. Some surveys recently completed in Indiana show that a large mileage of concrete pavements made with one particular aggregate always produced inferior concrete, regardless of the time of year of construction, the underlying subgrade soil, the traffic conditions, the type of cement used, the fine aggregate used, and any other variables that may have occurred in connection with a construction program involving this aggregate during the 12-yr period in which it was used. Other aggregates employed for short sections on these same contracts were invariably more satisfactory. Since the results of physical tests on such aggregates are not particularly alarming, there appears to be much need for the development of better tests for acceptance. As suggested by the author, the study of thin sections by a competent mineralogist might at least be helpful in determining the suitability of such aggregates in concrete construction. In this respect, the approach used by D. G. Runner (47) promises to give considerable assistance in solving such problems.

In regard to subgrade treatment and classification, a "pin-point perspective" seems to have been taken in many instances in correlating pavement performance with soil areas. In any given large geographical area, well-defined geological boundaries can be used to distinguish one soil area from another. Since the soil profile either is the product of disintegration and weathering of underlying bedrock or consists of transported materials primarily derived from rock,

it follows that similar soil profiles should be developed from similar materials provided the weathering has been of the same duration and that at least similar conditions of climate, topography, and vegetation have prevailed. From this it can be deduced that standard designs of the highway—particularly the base course, pavement, subgrade treatment, and drainage—can and should be made for each large uniform soil area. The only variable to be considered, then, would be the amount and weight of traffic loads that are to be anticipated. Many performance surveys made on flexible pavements in Indiana (48) lend support to this thesis.

Approximately 2,000 miles of secondary pavements in Indiana were surveyed during the 1943 spring breakup, yielding many interesting results. For instance, about 50 miles of traffic-bound gravel or stone bases with a bituminous top (total pavement thickness of 6 to 8 in.) located on gravel terraces suffered no noticeable ill effect from the combination of a severe winter and traffic. Similar pavements in silty clay soil areas suffered severely even with pavements as thick as 12 to 15 in. Another soil area in north-central Indiana consists of shallow sands on till, the sand being of variable thickness. Much pavement distress was encountered in this soil area, particularly where the shallow sand was 3 ft thick or less. Such an area may escape the engineer's attention since sand is normally an excellent subgrade material. However, a combination of traffic with adverse water and temperature conditions was sufficient during this particular spring to cause havoc with many miles of pavement, including some primary types. Obviously, a standardization of design with particular reference to grade and drainage is essential for proper pavement performance in this particular soil area.

Another pavement problem that can be studied on a geological-soil-area basis is that of pumping of rigid pavements. This action consists of the ejection of muddy water from beneath the pavements through cracks, joints, and along the edge of the pavement. Within a short time a void develops beneath the pavement, leaving the slab unsupported in certain places. Ultimately, cracking and complete disintegration of the pavement result (49). In the Middle West this problem prevails on heavily traveled roads located in lacustrine soil areas, such as in the Lake Chicago (northern Illinois and Indiana) and Lake Maumee (northern Ohio) soil deposits and in deep cuts in moraines. The extreme importance of the type of soil as a factor in pavement pumping cannot be overemphasized. The highly plastic coastal plain clays of Georgia, Alabama, Arkansas, and Texas, for example, produced badly pumping pavements where traffic loads are heavy. The engineer will be able to cope with this problem in the design of his pavement, but, since this new design may be more expensive than the older design, he should know where pumping pavements are likely to occur. The correlation of pavement performance with the geological-soil areas is a feasible approach to this type of problem.

Landslides may be studied frequently on a geological-area basis. The Conemaugh formation of the Pennsylvanian system, for instance, in some areas contains dozens of landslides, particularly in the "red-bed" outcrops. The Ordovician system outcropping in southeastern Indiana, southwestern Ohio, and north-central Kentucky is another landslide-producing geological area.

Corrections for landslides in such areas can almost be standardized, and certainly precautionary measures can be employed in the original road layout when it is realized that the geological strata in question have caused trouble repeatedly during and after previous construction operations. However, it should be emphasized that landslides will not necessarily prevail in other outcrops of Ordovician or Conemaugh rocks since this geological age classification does not require a similarity in rock textures.

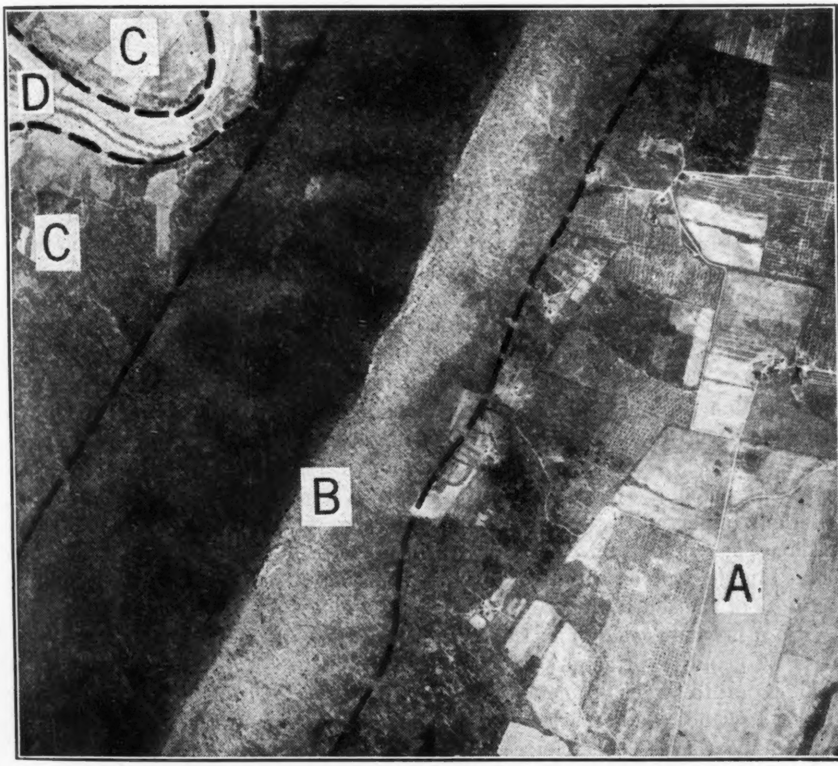

FIG. 4.—AIR PHOTOGRAPH OF A BEDROCK AREA—A IS LIMESTONE, B IS SANDSTONE, C IS SOFT SHALE, AND D IS RECENT RIVER ALLUVIUM

In correlating pavement performance with geological areas, one of the important weaknesses of geological literature is the stress that is placed on substantiating data showing similarity of age rather than similarity of texture of the particular strata in question. Age is important to the geologist in reconstructing the sequence of geological events. On the other hand, texture and composition of the rocks are of major importance to the engineer, particularly if he is to use geology in applying the results of experience gained in one area of the world to a similar area elsewhere. This difficulty is fast being overcome by the analysis of the patterns of aerial photographs. Experienced technicians can use such photos and predict with remarkable accuracy the

type of rock strata. A typical bedrock pattern is shown in Fig. 4. Even the inexperienced can see the contrasting patterns of the limestone, sandstone, and shale, designated as A, B, and C. The continued development and use of these techniques will require even greater coordination between the engineer and the geologist. In turn, active association of the two groups should do much to create a better understanding between them.

D. J. BELCHER,[14] JUN. AM. SOC. C. E.[14a]—The general thesis that geology is applicable to the solution of many highway engineering problems marks this paper as one to be read and considered by every engineer engaged in highway and airport engineering. It is hoped that some significance can be attached to this apparent trend toward a more diversified treatment of the various branches of endeavor classified as civil engineering.

The writer wishes to approach the discussion of this paper from a different direction—that of the engineer looking at geology. Without question, Mr. Huntting is representative of the progressive group of geologists that are contributing to the improvement of engineering practices in the construction field. That geology can contribute immensely to highway engineering is becoming more apparent each year. In fact, there are few, if any, who have explored the complete significance of geologic principles to these civil engineering problems.

The situation today is one in which the over-all work of the highway engineer is complicated by new and less tolerant standards. The influences and new problems should be met by recourse to available information and skills that may help to solve them. Unfortunately they are often unsolved. For example, many design engineers, field engineers, project engineers, and inspectors, who have opportunities to apply a knowledge of soils engineering, overlook opportunities to make corrections that will save time and money and prolong pavement life. Knowing the principles of soil formation, they can solve simple problems and recognize the potential problems that require the services of the soil engineer or soil scientist.

So it is with geology and the geologist; too few engineers realize the usefulness of geologic principles in everyday problems. Can they then see its application to larger problems?

As a science (and an art) geology is complex and requires a specialist's skill, but engineering is not geology and therefore does not often require the details or terms common in strictly geologic work. Having had the opportunity to examine geology in the light of engineering use in many of the states from east to west the writer has formed the personal conclusion that geology is very important in many areas and relatively unimportant in many others. Where it is important in the bedrock areas, the existing information and methods can be vastly simplified for their application to engineering work. Areas such as those covered by coastal plains sediments, great plains materials, glacial, and aeolian deposits are here considered separate from the general field of geology. Some difference in opinion may occur in the definition or scope of geology. However, the fact remains that soil forming processes in most of these areas

[14] Research Engr., Joint Highway Research Project, Purdue Univ., West Lafayette, Ind.
[14a] Received by the Secretary April 14, 1944.

create problems not amenable to geologic treatment. The official geologic map of the United States Geological Survey does not always concern itself with surface deposits but presents information on bedrock that is buried at depths of from 5 to 500 ft below the surface in the areas mentioned. Obviously the highway engineer is concerned with the character of the surface or near-surface materials.

The development of a second point is somewhat contingent upon the attitude taken on the scope of geology. If the field of the geologist is all inclusive, as it may well be by liberal definition, the situation is aggravated when one insists that a competent geologist is always essential. A competent geologist is a highly trained person whose talents may well be wasted on engineering problems in many states. Substitute instead a practical knowledge of geologic processes and principles in a number of the state highway department's engineering personnel and a greater potential improvement is in view.

Specialists can advance their field at a much greater rate by sharing their knowledge, acquainting others with their work, and thereby stimulating an interest in its application. The services of trained geologists will be in greater demand when there are more engineers with an interest in geology working in the field. That potential interest can be developed or retarded by the teachers of applied engineering geology in the colleges and universities.

If the average highway engineer does not use geology it is because he has not been shown the usefulness of the subject. The subject is not often deliberately and wilfully ignored. The fault lies largely with the geologists and the teachers of geology because they are not well acquainted with the problems of engineering. From this situation three conditions have developed:

(*a*) In their natural enthusiasm, the geologists have endeavored to apply the unmodified science to engineering problems;

(*b*) The situation, so common in many specialized fields, arises in which it is felt that only a highly trained geologist can deal with these problems; and

(*c*) The application of geology to highway engineering suffers when the specific engineering problems make the geologic concept seem both vague and general by comparison.

At this point it is well to make it clear that this commentary seeks first to advocate a greater application of geologic principles to the solution of highway problems to which they are pertinent, and then to add to and modify certain phases of Mr. Huntting's paper in the light of personal experience. To that end the following comments are offered:

Problem (1) Location of Pits and Quarries.—There can be no reasonable disagreement with the premise that sands and gravels as well as rock strata are predictable on the basis of a pattern or mode of occurrence that is in part geologic. These facts are seldom if ever taught in the classroom and it is indeed rare to find them presented to engineers in technical literature. The methods are simple and their mastery well within the capabilities of a competent materials engineer. There will be instances in which the special skills of the

geologist are necessary, especially in bedrock areas, but the bulk of the work will not require the specialist.

Problem (2) Suitability of Various Earth Materials for Surfacing, Concrete Construction, and Other Highway Uses.—The original presentation under this heading deals with a very significant phase of concrete aggregates that has a much more promising future than it has had a fruitful past. Unfortunately engineers have often chosen highly irrelevant test procedures as a means of testing aggregate quality for specific uses. The problems can be solved only when the origin, mineralogy, and chemistry of the aggregates are studied by competent persons. Obviously these skills can be found among the geologists, mineralogists, and chemists. It is suggested that this phase might be given additional consideration, whereas its inclusion with the less definite "earth materials * * * and other highway uses" to some extent detracts from the importance of the subject.

Problem (3) Subgrade Treatment and Classification and Problem (4) Frost-Heave Problems.—Since these are included in the writer's field of special endeavor, the manifest interest in Mr. Huntting's paper is understandable. These are problems that contain considerations other than geologic. In these two items the geologist may find himself far afield. The broad approach of the geologist has not demonstrated any appreciable suitability in dealing with problems of subgrade treatment or frost action despite the studies of crystal growth conducted by Professor Taber (15) and the resulting contributions to the knowledge of frost action. In the solution of field problems, frost action is a complex problem. It occurs as a product of the whims of ground water as well as of soil texture and is found more often in sands (shallow deposits on till), silty sands, and silts, rather than in clays as indicated.

The writer feels that ultimately a geologic method of classification suitable for engineering usage will form the basis of soil classification, but to date it has not been fully developed. When offered initially, engineers in general may express little sympathy for such a classification because of their classroom impressions of geology.

Returning to the geologist's present method of soil classification—residual, colluvial, organic, transported—it may be cited as an example of unrelated standards of classification. In terms of engineering problems this method lacks realism because:

Residual soils range in texture from the common clays (limestone, granite, basalt, etc.) through the silts, sands, and sandy clays (sandstones, schists, granites, etc.) to residual gravels.

Colluvial soils occupy relatively insignificant areas, vary so widely in texture, lack uniformity, occur on steep slopes, and are so highly susceptible to wide variations in water conditions that little of engineering significance can be said in the general terms necessary to a classification.

Organic soils, all significantly associated with a high water table, are worthy of distinction as a separate class and a further subdivision should probably differentiate between organic-sand mixtures of the glacial outwash areas and the mucks, peat, and muskeg swamps.

The term, "transported soils," like other classifications, is broad and includes all textures and all ground-water conditions. To illustrate:

An alluvial soil area, for example Mississippi River alluvium, varies in texture from medium sand in Minnesota through silts in Illinois to clays from Missouri to the Gulf. At any one point in the flood plains (south of the Wisconsin State border) one may expect sand on the natural levee and silt or plastic clay in the "slack water" areas. Since these alluvial soils vary in texture with the size of the stream and the nature of the soil in the watershed, the method of classification cannot be too general and remain cogent.

Lacustrine soils are uniform by comparison since they are generally of silt or silty-clay texture, whereas marine sediments (of the coastal plains) provide highly plastic clays as well as sands and silts. Likewise glacial soil as a classification embraces the plastic silty clays of the till plains, sands of outwash plains, and gravels of the valley plains, kames, and eskers.

The aeolian group, if subdivided into sand and loess, provides two remarkably uniform soil materials.

It has been acknowledged (see heading, "Subgrade Treatment and Classification") that "The soils falling under a given heading in this classification will not have all their physical properties alike * * *" and that, "Nothing can replace the engineers' physical tests * * *." Physical tests show similarity among the vast areas of lacustrine silt in the glacial lakes Agassiz, North Dakota and Minnesota, Maumee, northern Ohio, Lahonton, Utah, and Bonneville, Washington; in the loess in Mississippi, Tennessee, Illinois, Minnesota, Iowa, Indiana, Kansas, Missouri, Nebraska, Idaho, Washington, and North China—to cite a few; and in the alluvium in Ohio, Indiana, Illinois, and Iowa; and some residual soils derived from fine-grained sandstone. Obviously the engineering problems in these different areas are not all similar, so physical tests alone are often misleading.

This is more a criticism of engineers relying on test data than of the geologic approach, and the geologist will more readily understand it. The engineer relying on test data alone often finds himself standing on one leg (technically and figuratively), and his designs are continually off-balance because other conditions upon which good performance is contingent do not appear in design formulas.

If the geologist's approach is too broad and the contemporary engineer's too narrow, the solution may then be found in a blending that can be obtained by a better mutual understanding. Add geologic principles to highway (soils and materials) engineering. Combine test data and the soil position or land form. Consider the natural position, the soil structure, the profile horizons, the drainage, and the climate. The solution lies largely in the hands of the geologist-engineer-professor—not the geologist, not the engineer, not the professor.

Problem (5) Prediction of Character of Material To Be Excavated.—This is an important phase of highway location and planning. Many engineers

familiar with a few fundamentals of rock weathering and geology react unfavorably to the implication that a competent geologist is always required to perform these services.

In the "Prevention and Correction of Landslides," Problem (6), the geologist can render his most valuable service in many afflicted areas. In rapid succession all can approve the listing of his value in tunneling operations, Problem (9); evaluation of mineral lands, Problem (11); and as an expert witness in related court actions, Problem (12). However, the cause suffers when the geologist presumes to render comparable service for highway engineers except in special cases in judging bridge foundations, Problem (7); predicting channel changes, Problem (8); and underground-water supplies, Problem (10). In each of these it is felt that the descriptions offered do not substantiate the positive character of the statement of the problem.

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AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

MILITARY AIRFIELDS
A SYMPOSIUM

Discussion

BY WILLIAM E. RUDOLPH, RAYMOND L. IRWIN, G. G. GREULICH, HIBBERT HILL, JACOB FELD, AND ROBERT HORONJEFF

WILLIAM E. RUDOLPH,[9] M. AM. SOC. C. E.[9a]—The engineer who has followed the development in airplane landing fields must be impressed by the thorough manner in which the Corps of Engineers has solved the two important problems of paving and drainage, as described in this Symposium.

Colonel Stratton outlines the wheel loadings (as high as 60,000 lb) on which designs of pavements for military airfields are being based. Under the heading, "Loading Criteria for Military Airfields," he states: "Loadings greater than 60,000 lb may be anticipated in the near future, but no design provision is being made yet for such loadings." The writer would raise the question: How will the pavements designed for present loadings be converted to future loadings, when and if such increased loadings are adopted? Recent news items indicate that Idlewild Airport in New York, N. Y., may have runways designed for 300,000-lb plane loads—wheel loads more than double those for Class I airfield pavements. Thus, another and even more important question applying to lengths of runways as well as loadings should be asked: When and where is the limit to be established on increases in design requirements for airfields?

Somewhere the writer has read that development in airplanes generally has been ahead of airport and airport facilities. This statement is doubtless true, but for a reason. After all, the civil engineer has had the continuous task of keeping up with the airplane designer, providing airports and facilities for the landing and the taking off of larger and heavier types of aircraft at ever-increasing landing speeds. Because early aircraft had no brakes, the airfield builder had to provide a large number of runways for landings into prevailing

NOTE.—This Symposium was published in January, 1944, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: February, 1944, by James B. Newman, Jr.; March, 1944, by Thomas E. Stanton; and April, 1944, by W. E. Howland, and David S. Jenkins.

[9] Eng. Dept., Anaconda Copper Mining Co., New York, N. Y.

[9a] Received by the Secretary March 23, 1944.

winds. As wing loads and operating speeds increased, he had to lengthen these runways to provide for longer take-offs and landing runs. The resulting field became more and more cumbersome. As wheel loads reached the stage at which turf fields were no longer appropriate, the engineer had to give more attention to soil stability, paved strips, and expensive drainage systems. It is small wonder that throughout the history of aviation, after several years of service, landing fields became obsolescent—too small for the kind as well as amount of air traffic using them.

Will the airfield continue to be laid out and built to fit the airplane, or will the airplane designer build his craft to fit the airfield? Economic laws will dictate the answer to this question. Already engineers are beginning to stress limitations on the size of an airport rather than increases in runway lengths and loadings. The development of catapults for aid in take-offs and of tricycle landing gear for cross-wind landings, as well as of better air and ground brakes, promises to keep the lengths of runways within the limits mentioned by Colonel Stratton; however, wheel loads might be destined to increase further as the cargo plane replaces the bomber in the postwar world.

Colonel Stratton mentions the current rule by which runway lengths are increased 500 ft for every 1,000 ft above sea level. This necessity seldom involves important increases in airfield dimensions within the United States, but it does create problems in South America, where the mining regions and their population centers are generally located at altitudes above 7,000 ft (many above 12,000 ft). South America is far more dependent on air services for cargo, not to mention passenger and mail, transport than is the United States. Its railways and highways have not been developed to any extent in most sections, and the obstacles of rugged topography and rainy season stream flows make construction very expensive. The European interests which earlier played an important part in developing South American aviation have now been removed, and the engineers of the United States face the task of furnishing the "know how" in completing this development, particularly as applied to postwar cargo transport. The cost of landing fields with the extremely long runways required at high altitudes, located necessarily in regions of rugged topography, presents a problem which must be solved by the civil engineer and the airplane designer working together. The writer recalls an experience with airfield location in Bolivia about 1936, when serving on a committee entrusted with the task of recommending an airfield site near the City of Potosi. Two sites appeared appropriate, one at an elevation slightly below 13,000 ft, the other at one somewhat above 11,000 ft. The higher site had greater advantages—or, more exactly, fewer natural disadvantages—than did the lower, but the army officer making the final decision was interested only in the lower site. Here the additional runway length required at the higher elevation, although relatively small, was the governing factor against this location, available space within the narrow valley being scant for take-off and landing requirements in either case.

A consideration not mentioned by Colonel Stratton in connection with the choice of airfield site is concealment. The writer has read with interest a

recent book (35)[9b] on this subject by Merrill E. De Longe, in which it is stated that hard surface runways are invariably the most conspicuous features of an air base. Undoubtedly the Corps of Engineers has given serious consideration to reducing such runway visibility.

RAYMOND L. IRWIN,[10] Assoc. M. AM. Soc. C. E.[10a]—The procedures recommended in this Symposium by Mr. Hathaway for the design of airfield drainage facilities are impressive. The applications of the more advanced theories on drainage, particularly those concerning infiltration and surface runoff, are, in themselves, valuable contributions toward future drainage studies and computations.

There is an apparent need for further research to determine the basic data, which play so important a part in the development of the computations and curves. In this connection, attention is invited to the following items which are ordinarily considered essentials in drainage studies and designs: (*a*) Rainfall intensity and frequency data; (*b*) procedures for the determination of rainfall infiltration rates; (*c*) effects of surface detention storage; and (*d*) roughness coefficients applicable to anticipated grass cover.

Available records show that the runoff resulting from an excessive storm is not usually in proportion to experienced rainfall intensities. An intense rainfall which normally occurs during the summer season ordinarily provides less runoff from a given area than a storm of corresponding intensity which occurs during the winter season. There is also a perceptible difference in the intensities of rainfalls with seasonal changes. A comparison of rainfall frequency-intensity data taken for the entire year, as against frequencies and intensities experienced for the six months, November to April, was made by the writer on the basis of data obtained from rainfall records for Pittsburgh, Pa., Cincinnati, Ohio, and Cairo, Ill. For each of these stations, at least seventeen summer rainfall occurrences of 1-hr duration have exceeded the highest winter rainfall intensity for one hour, during the 35-yr period from 1905 to 1939, inclusive. It is an accepted fact also that infiltration rates for summer rainfall occurrences should ordinarily be considered greater than those for storms occurring during the winter seasons. Since infiltration rates and rainfall intensities vary with the seasons of the year, recognition of these factors might be contained in the construction of runoff frequency curves.

The values of runoff used for construction of the curves might be obtained by subtracting the infiltration rate to be applied during a given period, say, a month, from the intensity of the rainfall to be expected during that month. The data could then be combined on a yearly basis if so desired. In this manner it would be possible to refer to the frequency of rainfall contributing to runoff, which is the factor more indicative of the capacity for which protection and design are desired.

[9b] Numerals in parentheses, thus: (35), refer to corresponding items in the Bibliography at the end of the Symposium, and at the end of discussion in this issue.

[10] Engr., U. S. Engr. Office, Columbus, Ohio.

[10a] Received by the Secretary March 29, 1944.

The methods for determining infiltration rates have not advanced sufficiently to furnish reliable values for use in design. After establishing infiltration rates based upon careful tests and studies, it has been found by experience that soils which appear porous sometimes are quite susceptible to clogging from the fines and vegetable matter contained in top soil and transported by the surface runoff. This action has been noted to occur particularly where the ground surface is disturbed by grading operations, prior to the establishment of a protective turf covering. The results of such experiences indicate the advisability, even for soils with high infiltration rates, of providing pipe or ditch drainage systems capable, eventually, of disposing of apparently unforeseen surface ponding.

The problem of grading to specified take-off and landing grades for all-over type airfields, although apparently more desirable for plane operations, often has the disadvantage of creating areas that result in surface detention pockets. This is usually caused by uneven settlement of ground in areas where slight slopes are constructed. In most cases, these areas are not harmful in themselves, although basic formulas utilized in drainage computations are disturbed. It seems unreasonable to demand that a contractor maintain surface grades in agreement with theoretical grades within as much as 0.1 ft or 0.2 ft. For fields where grading was considered quite satisfactory, topographic surveys have indicated the existence of these irregularities, which usually have no effect upon plane operations. On steeper slopes care should be exercised in maintaining the proper grades. Slight differences in slope may tend to collect the flow along a given line. This condition might result in channeling, which will tend toward higher velocities and erosion. As a consequence, shorter concentration periods will be applicable. If large ponding areas are involved in the process of overland flow, the concentration time of so-called overland flow apparently would be reduced because of the nearly negligible friction coefficient involved in traversing the ponded areas. Therefore, a greater peak outflow would result.

Various types of soils are encountered in airport construction. Some of these are more responsive to the growth of turf than others. To avoid unnecessary expense in treating the soil, types of grass are grown on some fields which are not practical for other fields. It has been suggested that, if the ground will not grow grass, weeds should be grown to obtain a protective covering. The writer is not familiar with any specific project in which the planting of weeds was found necessary.

For the various conditions of grass cover, which differ widely in structure and texture, difficulty will occur in the evaluation of the roughness coefficients. The Manning formula, used for determining flow, indicates that, for a given depth, the discharge varies inversely with the roughness factor. The roughness factors recommended by Mr. Hathaway range from 0.2 to 0.8. The corresponding discharge values for a given depth, other factors remaining constant, would therefore have the ratio of 4 to 1. Roughness coefficients are frequently selected under the guidance of judgment and experience. Therefore, the values selected might be considered questionable until sufficient supporting data and a dependable procedure are made available for practical application.

Experiments by W. O. Ree (36) on shallow flows over grassed slopes indicate that, after depths of flow of 2.5 in. were exceeded with certain grass cover, the roughness coefficient tended to decrease with increase in depth. At a depth of approximately 5 in., the roughness was less than 80% of that at 2.5 in. (Since runoff velocities for grass are very low, the aforementioned depths are feasible for long slopes.) The change in roughness may be caused by arrangement of the grass blades or reduction of the wetted perimeter by the force and weight of the water. A finer type of grass apparently will be quite sensitive and will cause appreciable changes in discharge at shallower depths. Here again experimental information seems necessary before adequate data can be secured to permit accurate reproduction of profiles for overland flow.

The rates of overland flow corresponding to standard supply curves, Fig. 19, do not represent the absolute maximum rate of runoff for a storm of a given frequency if the rainfall sequence is disturbed. Mr. Hathaway states that the curves of Fig. 19 are not hydrographs, but represent the peak rates of runoff from individual storm events of various durations, all of which have the same average frequency of occurrence. At the beginning of the design storm, which is of uniform rate, the ground surface is free from flow; therefore, storage for overland flow does not exist. This does not represent the usual condition experienced preceding high rainfall intensities. Runoff occurring from the lighter rainfall immediately before the design intensities will often displace a part of the surface detention storage.

In the interest of determining the maximum rate of runoff that could be obtained for a hypothetical storm containing runoff intensity values corresponding to a given supply curve, the writer made the following analysis: Rainfall intensity values corrected for infiltration were obtained from the standard supply curve 1.6 for 5-min periods, and their values were arranged in ascending order. These values were routed graphically through the storage occupied in overland flow, by the method suggested by W. W. Horner, M. Am. Soc. C. E., and S. W. Jens, Assoc. M. Am. Soc. C. E. (24), using the relation $q = K y^2$. In brief, on the rising side of the hydrograph, the volumes of surface detention or storage for two different but uniform net rainfalls are considered equal, when the runoff rates are equal. Therefore, when changing from a lower to a higher rainfall rate, the discharge rate obtained at the end of the time for the first selected rainfall intensity is proportional to the storage and can be transferred to apply, at an equal discharge rate, to the second rainfall supply curve. This curve is then paralleled for the desired interval of time to obtain the runoff at the end of time for period two. Fig. 30 indicates the steps taken for routing with $L = 400$ ft and the intensity values obtained from the 1.6 supply curve. The maximum runoff rate obtained by this method was 2.9 in. per hr as compared to a maximum value of 1.75 in. per hr, furnished by Mr. Hathaway's procedure. Fig. 19 indicates a maximum runoff rate of 2.3 in. per hr for supply curve No. 2.0 and $L = 400$ ft. Although the former value of 2.9 in. per hr represents an extreme condition, but not necessarily the result of a true frequency, it was presented to demonstrate the wide deviation

in results possible from the same indicated frequency storm with a different arrangement of the rainfall. Since storms of lower intensity occur more frequently, they should produce a runoff of, say, 1.75 in. per hr where the true frequency would apparently be less than indicated by Mr. Hathaway. There is a greater possibility that the runoff intensities are higher for the shorter lengths of overland flow than for conditions involving travel over the longer lengths. The extreme for minimum flow could be reached by reversing the rainfall distribution, under which condition a value of approximately 1.2 in.

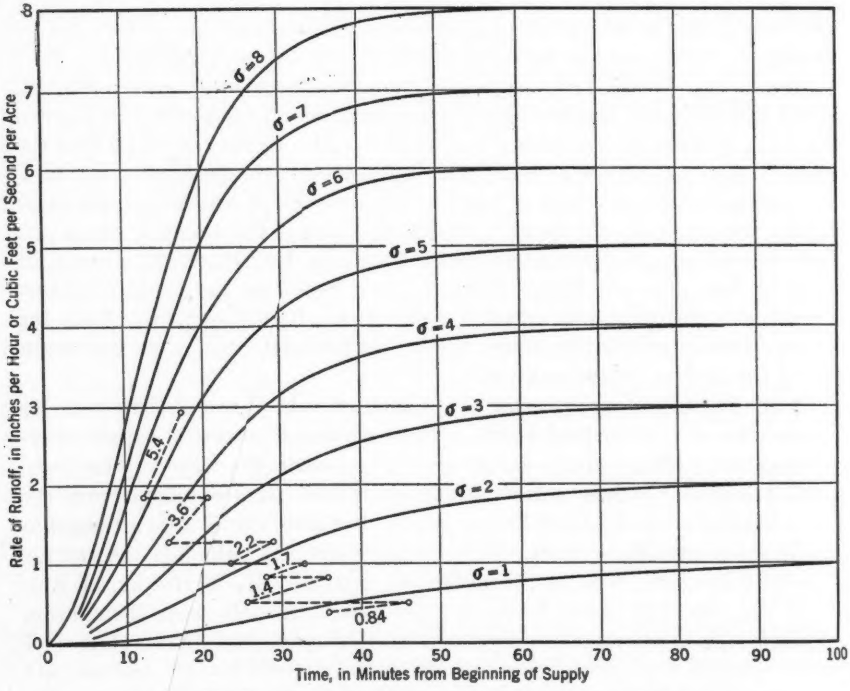

FIG. 30.—ROUTING WITH INCREASING EFFECTIVE RAINFALL RATES

per hr would obtain. A careful study of critical rainfall distribution deserves investigation to determine critical conditions for the area under consideration.

A graphical solution used by the writer to determine storage and drain-inlet capacity may be of interest. Where the storage is very small, the rainfall distribution is arranged for maximum conditions as stated in the foregoing paragraph, using descending values of intensities. The method proposed is approximate only and consists essentially of the following:

(*a*) Construct a mass curve of rainfall versus time corrected for infiltration rates (see Fig. 31). The rainfall intensities for short intervals of time are taken from the supply curve of Fig. 15 and are arranged in descending order to obtain minimum overland flow storage at the time of maximum ponding.

(*b*) An outflow hydrograph is constructed in accordance with the method previously outlined for the rising side of the hydrograph. When the outflow reaches a peak by this procedure, it is assumed that the outflow or net runoff for the recession side of the hydrograph is equal to the rainfall rate minus the infiltration rate.

(*c*) The storage for overland flow is then obtained from the relation $q = K^2$, in which the symbols are the same as in the Symposium.

(*d*) The instantaneous storage produced by overland flow is subtracted from the mass rainfall. (The outflow hydrograph was not constructed to balance the inflow graph.) The procedure gives a mass runoff curve which is somewhat conservative.

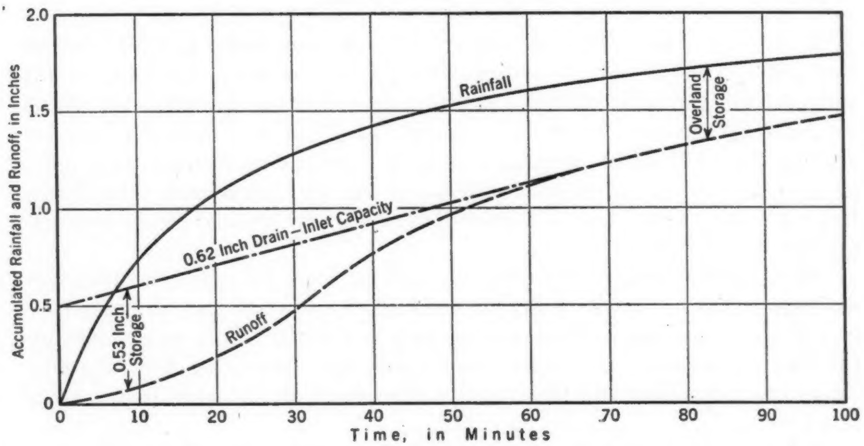

FIG. 31.—GRAPHICAL SOLUTION FOR DETERMINATION OF DRAIN-INLET CAPACITY, FOR STANDARD SUPPLY = 1.6; L = 400 FT

(*e*) By the method of rate lines similar to those used for the solution of water-supply storage problems, the slope of the tangent to the mass curve indicates the drain-inlet capacity. The required storage value is given by the greatest vertical distance from the rate line to the runoff curve.

If accepted, the overland flow curves developed by Mr. Hathaway may be used advantageously in combination with the well-known rational method to determine the flow not affected by ponding. A differential of 10 min or even 20 min in the application of the overland flow curves for the longer lengths of travel is indicated by this method to differ only slightly from values obtained by the procedure developed by Mr. Hathaway. This is true provided the error is on the long side of the time or somewhat past the t_l-value. For the 1.6 supply curve and lengths ranging from 150 to 600 ft, an error of 20 min will not affect the results of computations by more than 1·6%. In view of this fact, the flow at junction chambers might be obtained accurately enough by direct addition of the critical branch-line values, with adjustments for runoff from paved areas, provided that the entire drainage system is relatively short. In

terms of distance a 10-min interval considered for time of flow through a closed conduit, with an average velocity assumed as 3.3 ft per sec, would constitute a travel distance of 2,000 ft. Following this procedure with consideration for junctions of long lines the design capacities can be computed or checked quite rapidly.

At some of the more recently constructed airfields, recording raingraph and recording hydrograph instruments have been installed to furnish accurate records and to present a sound basis for comparing design values with those actually obtained in nature. It will be quite some time before sufficient data are collected, and before satisfactory explanations concerning runoff can be attempted. However, strides are being made in what appears to be the proper direction toward better design for airfield drainage.

G. G. Greulich,[11] M. Am. Soc. C. E.[11a]—As repeatedly indicated in this Symposium, the effects of moisture changes have a great influence on the subgrade soil and base courses. Moisture accumulation, in particular, affects subgrade deformation, remolding, pumping at joints, and density or volume changes due to frost, heaving, and thawing. Deterioration from these influences makes the determination of subgrade ratings by plate bearing or California Bearing Ratio (CBR) tests a matter of guesswork where year-round performance predictions are necessary.

These factors seem to indicate the need for more attention to subsurface drainage systems. Lowering the ground-water level by adequate drainage may give greatly improved performance at much less cost than can be obtained by thickening pavements and base courses and by compacting the subgrade. Every one is familiar with the crushing strength of dry, as compared to saturated, soils.

Accumulation or ponding of water during construction should be prevented as much as possible. Water seeping through pavement joints should be able to flow readily to subgrade drains. This will prevent excessive moisture accumulations in the soil at points where the surface slabs are weakest and where the greatest relative vertical movement of adjacent slab edges and attendant reworking or churning may result.

In mountainous or foothill country, and even in gently sloping plain country, there may be subsurface flow or seepage of ground water which normally does not approach the surface near enough to affect stability or carrying capacity of the upper soil layers. However, during wet seasons, this flow may occur very near to, or even emerge at, the surface. Under such conditions, during or immediately following the wet season, subsurface drainage will take care of most of such flow and prevent serious changes in the moisture content of the subgrade soil.

Under the preceding conditions, the most excellently designed and functioning surface drainage systems cannot prevent saturation from subsurface flow originating at points that may be remote from the airfield. Infiltration

[11] Cons. Engr., Specialty Div., Sales Dept., Carnegie-Illinois Steel Corp., Pittsburgh, Pa.

[11a] Received by the Secretary April 4, 1944.

of water in surrounding areas may also accelerate flow under the surface of the airfield.

Whereas inadequate surface drainage may hamper operations only for a limited time, inadequate subsurface drainage may reduce the usefulness of a field for a long period and may even render it completely unserviceable, particularly as more planes with Class I wheel loading come into service.

An extension of the subject of subgrade drainage in the closing discussions would provide a most welcome contribution to general engineering knowledge on the design of military airfields.

HIBBERT HILL,[12] ASSOC. M. AM. SOC. C. E.[12a]—The influence of load repetition on the life of pavements is emphasized by Colonel Stratton. This effect is very evident in the accelerated traffic tests described in his paper. In these tests a pavement characteristically will show no distress whatever during the first passages of the load, and yet may fail completely (and progressively) after a relatively small number of passages. The same phenomenon is a matter of common experience to paving engineers, although not usually so readily observed in practice, because of the length of time involved, as in tests.

The accelerated traffic tests progress from the condition of newly completed construction to total failure in a matter of days or weeks. The condition of the pavement base and subgrade is thus substantially "as built" during the course of the test. On the other hand, perhaps a runway will not be subjected to an equal number of stress repetitions for years. During this time, and in addition to other time effects, moisture conditions in the base and subgrade may have changed, frost may have acted on the base and subgrade, and the paving itself may have deteriorated because of factors other than load stresses.

Thus, in translating accelerated test data into terms of runway pavement life, two questions at once arise: (1) To what actual number of stress repetitions will the runway pavement be in fact subjected? (2) What is the relation between the effects of rapid repetitions of load in an accelerated test and the effects of the much more widely spaced (in time) repetitions to which a runway will be subjected in operation?

Existing data and calculations clarify the first question considerably. The second question can be disposed of rapidly. The answer is not known at the present time. It can only be assumed, on the basis of general experience, that repetitions in one case are approximately equal in effect to repetitions in the second case—that is, if under accelerated conditions a pavement fails at the end of one thousand load repetitions taking place in say ten days, it will also fail at the end of one thousand load repetitions spaced over a much longer period.

In accelerated traffic tests, a load is made to pass repeatedly over a lane of given width in such a manner that the wheel tracks uniformly cover the surface of the lane; for example, assuming a 5-ft lane and a tire imprint width of 1 ft, the first passage is in the 1-ft strip along one edge of the lane, the second passage is in the strip between 1 ft and 2 ft from the edge of the lane, and so on, until, in five passages of the wheel, the entire lane has been covered. If the tire imprint

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[12] Lt. Col., Corps of Engrs., Deputy Chf., Eng. & Development Div., Office, Chf. of Engrs., War Dept., Washington, D. C.

[12a] Received by the Secretary April 5, 1944.

is narrower or wider than this example, a greater or lesser number of passages is necessary to cover the lane. Such a series of passages has been regarded, in the tests, as one coverage, or one maximum load repetition. Load repetitions are reported in terms of number of coverages in lanes of a given width.

Thus, if, in the case of an operating runway, a longitudinal element of the 10-ft-wide runway is considered and it is known that this 10-ft element has been traversed one hundred times by a wheel with a 6-in.-wide footprint, then it can be assumed that the 10-ft strip has experienced five coverages, or five maximum load repetitions. Such an assumption is only valid if the 10-ft strip can be considered as a small element of the entire width. The same assumption could not be properly made in the case of a taxiway, where the narrow pavement would force traffic into narrower lanes.

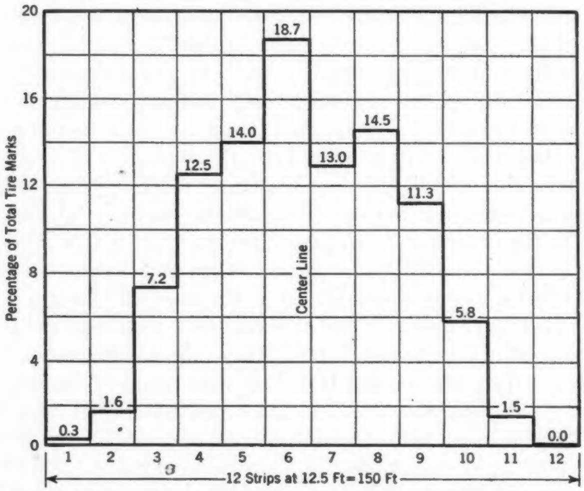

FIG. 32.—PERCENTAGE OF TOTAL TIRE MARKS IN EACH STRIP WITHOUT RESPECT TO LONGITUDINAL DISTRIBUTION

The usual military runway is 150 ft wide. On such a runway it may be assumed that the pilot will plan to land the center line of the plane on the runway center line and that deviations from this plan will be accidental in a statistical sense. Thus, the distribution of landings over the runway width will approximately conform to the normal law of error, and the maximum number of coverages will be on the central element of the runway.

A study of the possibilities leads to the conclusion that from 16% to 24% of the landings (center line of the plane) will be on the center 12.5-ft strip of a 150-ft-wide runway if there are no constant factors tending to channelize the traffic. Fig. 32 shows the actual distribution of landings on a particular runway used exclusively by fighter planes. These data were obtained by counting tire marks on a concrete runway after about six months of service. Since it is presumed that there are two tire marks per landing, the percentages of landings and of the tire marks are approximately equal. Actually, the tire marks may exceed twice the number of landings, since the plane may bounce in a poor landing, leaving two or more sets of marks.

The total number of tire marks counted to obtain Fig. 32 was 2,010. Of these, 1,132 marks were counted at the south end of the runway and 878 marks at the north end. Longitudinally (Fig. 33) the marks are rather well distributed over about 1,500 ft of each end of the runway, but with a maximum point about 1,000 ft from the end.

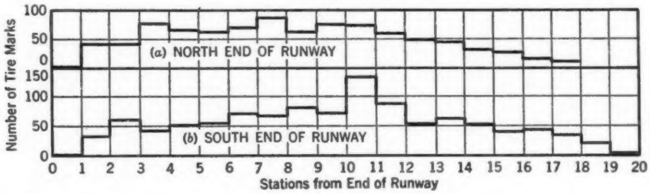

FIG. 33.—NUMBER OF TIRE MARKS IN EACH TRANSVERSE SECTION BETWEEN STATIONS

Each landing loads two points on the pavement, one point under each wheel (the nose or tail wheel load is small and may be neglected). The total number of load repetitions is therefore twice the number of landings. Fig. 34 represents

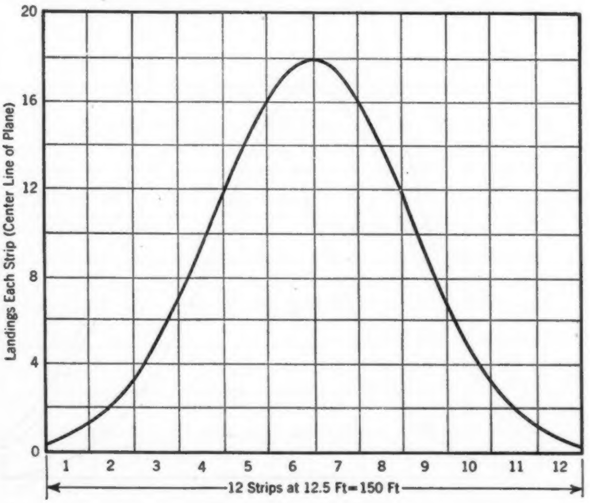

FIG. 34.—DISTRIBUTION OF ONE HUNDRED LANDINGS

one hundred landings distributed over a 150-ft-wide runway in accordance with the normal law of error,

$$y = K e^{-h^2 x^2} \qquad (10)$$

in which $h = 0.1$. The curve represents the locations of the center line of the plane during the various landings. The wheels of the plane are displaced from its center line by an amount in general dependent upon the size of the plane. In Fig. 35, this displacement is assumed to be 12.5 ft (25-ft spread between wheels), representing a large plane. The number of wheel passages in a given strip then is the sum of the ordinates of the two curves in Fig. 35, representing

the individual wheel positions. The maximum ordinate of the resultant curve is less than twice the maximum ordinate of the base curves, varying with the magnitude of the wheel spread. This difference increases as the wheel spread increases and is in general significant only for the largest planes. In the example shown (Fig. 35) the maximum ordinate of the resultant curve is 10% less than twice the maximum ordinate of the base curve.

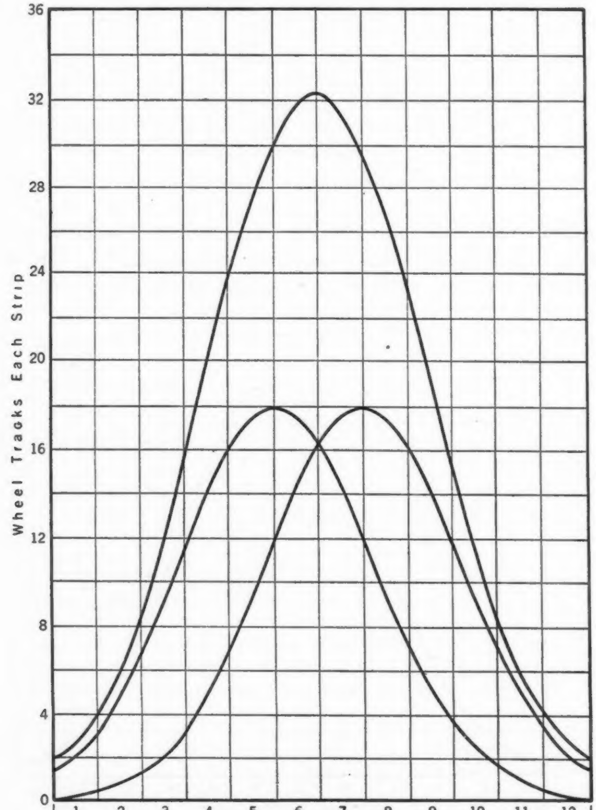

Fig. 35.—Distribution of Wheel Tracks; One Hundred Landings

The most frequently loaded strip will be at the center of the runway, and, for practical purposes, the number of passages over a given width of center strip will be twice the number of landings on that strip. The number of load repetitions (coverages in the sense of this discussion) will be the number of passages over the strip multiplied by the tire imprint width and divided by the width of the strip considered. The latter factor is of great importance. The width of the imprint of a fighter plane is about 8 in., and that of a bomber is about 16 in. The bomber, therefore, occasions about twice the number of load repetitions that the fighter causes during the same number of landings.

This discussion relates entirely to landings. Similar considerations will apply to take-offs. Probably take-offs, being more closely under the control of the pilot as to location on the runway, will tend to channelize to a greater extent than do landings. Other factors being equal, the take-off will be approximately along the center of the runway. However, the latter statement would not apply to wide runways; nor does it apply to small planes, which will tend to take off along the side of the runway that is first reached from the taxiway. Thus, local conditions, such as the location of the taxiway exits, will tend to channelize traffic on take-off. Fighters taking off in formation will deliberately use the sides of the runways. At night the pilot will follow the side (left side for large planes) of the runway where he can guide himself by the runway marker lights.

A plane on landing ordinarily first touches the pavement within the first thousand feet of runway. At this point, and for perhaps 1,000 ft beyond, the plane is substantially air-borne, and therefore does not cause maximum load on the runway. The maximum load on the pavement will not be felt until the speed has reduced to that of taxying, and this load will be imposed on a length of runway dependent on the location of the exit taxiway. Similarly, a plane taking off will be substantially air-borne after about 1,000 ft of run. Therefore, the take-offs cause maximum stress to the runway only at the end of the runway from which they occur and the landings only at the opposite end. Each end thus receives 50% of the stress repetitions due to the total operations, regardless of the wind rose. This can be seen readily in the extreme where all operations are in one direction. In such a case one end of the runway will be stressed to the maximum by every landing, but by no take-off, whereas the opposite end will be stressed by every take-off, but by no landing. Dependent, again, on the field layout and on the conduct of local operations, a central part of the runway will receive only accidental and occasionally maximum stresses.

The foregoing considerations have not much influenced airfield pavement design to the present time. As Colonel Stratton states, however, designs of the Corps of Engineers have been influenced to the extent that heavier pavement is provided on taxiways and on aprons (where taxying or standing traffic is channelized) than on the runways proper. The Corps of Engineers considers the effect of repetitions on the life of the pavement in designing advanced airfields for temporary operation. Since such considerations may have considerable economic importance in connection with the heavy construction required for present and future large airplanes, they should receive increasing attention.

Jacob Feld,[13] M. Am. Soc. C. E.[13a]—Solutions for practically every design and construction problem which arises in the performance not only of military but of commercial airfields are given by Colonel Stratton in this Symposium. A few comments from the construction point of view are made not as a criticism of the material in the paper but as a possible explanation for the manner in which the design problems have been solved within the limitation of construction feasibility.

[13] Cons. Engr., New York, N. Y.

[13a] Received by the Secretary April 12, 1944.

In standard sections of concrete pavement, most of which (because of the priority regulations affecting the use of steel reinforcement) are practically devoid of any steel, the thickening of edges has introduced difficulties. Construction of an airfield is always a large-scale operation. The preparation of the subgrade and the application and the compaction of the sub-base—both must be done in large areas if uniform results are expected and if the work is to be done economically. The requirement that the subgrade of the concrete be shaped to provide for thickened edges, especially if the thickened edges occur adjacent to the forms, introduces a new operation which disturbs the compacted subgrade. If the compacted material is removed by the blade of a power unit or even if it is removed by hand, the area must be extended sufficiently to provide space for setting up forms. It is impossible to cut away a wedge-sectioned strip of a compacted sub-base without some disturbance. It is almost impossible to reroll or recompact the irregular shaped subgrade to a density equal to that of the undisturbed adjacent material. In addition, when the concrete is being placed, the vibration required adjacent to the forms to eliminate honeycomb further disturbs the density of the subgrade. Such disturbance may be the cause of honeycomb often found in concrete in the vicinity of the forms, since the reduced density of the subgrade tends to absorb and remove the mortar from the concrete placed on it. Except, of course, for the necessity for hand work in shaping the subgrade, the thickened edges at joints transverse to the direction of the forms are less objectionable from the point of view of the aforementioned difficulties.

In scheduling the concrete pavement, there is always a difference in opinion as to sequence of lane pouring. Alternate lane construction has the advantage that it releases the forms faster, but as Colonel Stratton states it has some serious disadvantages because of the disturbed subgrade and the possibility of ponding of rain water between completed strips. These objections are overcome by pouring around completed strips, and, if the length of the strip is properly correlated with daily progress, a satisfactory schedule can be maintained. By permitting a time gap after the first strip, used to fill in other concrete operations, continuous operation in placing additional strips can be scheduled by adding strips to the first one poured in a continuous and circular sequence. If the length of the strip is made equal to a 2-day capacity of the unit used for placing concrete, no time loss will occur. After the second strip is started (the concrete being properly set up to support the finishing machine), the operation is away from the completed concrete at all times, and subgrade preparation is never fenced in by completed work.

Under the heading, "Pavement Design for Frost Action," Colonel Stratton states: "* * * the combined thickness of pavement and base material not subject to frost action is made equal to the average depth of frost penetration as determined from local records." This statement should be modified by considering the different heat transmission coefficients of materials like coarse sand or gravel which are ordinarily used for base course, as against the much finer and more easily saturated local materials displaced by the base course.

The writer has noticed that the compaction of added base-course material to specified densities, greater than the natural densities of the immediately

underlying material, presents a temporary condition of unbalance which proves a weakness under the impact of traffic. Sudden change in density within the soil body seems contrary to natural conditions and should not be specified. The tendency to readjust density with time is aggravated by traffic vibration when adjacent horizontal layers are of different densities. There is a very similar, and to some extent more serious, adjustment when trenches dug in the natural soil are backfilled to a specified density greater than the density of the undisturbed soil. Such backfilled drainage ditches form a network of relatively hard intrusions in the natural soil condition and almost always cause trouble in the finished pavement.

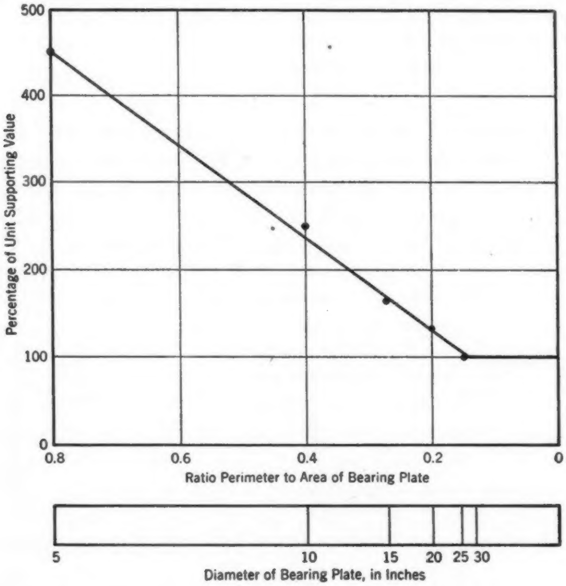

FIG. 36

Fig. 36 is plotted from data taken from Fig. 5, and shows a linear correlation between percentage of unit supporting value and the ratio "perimeter: area" of the test bearing plates smaller than 30 in. in diameter.

In the part of Colonel Stratton's paper on drainage, the entire discussion of subsurface drainage is based on the assumption that the pavement is perfectly impervious. If any rain water seeps through the joints or through cracks into the sub-base or if the underlying base course is saturated and at times possibly below ground-water level, a positive subsurface drainage should be provided. In installations where a porous sub-base is installed on an impervious soil, ponding-of-water accumulation in the sub-base would cause failure of the pavement by its effect on the subgrade material.

The system shown in Fig. 8(*b*) for the base-course drainage is a specified finished product which, under the usual construction schedules, is impossible to build. Analyzing the various stages of the work, such an installation would require:

Either—	or—
1. General excavation to the top of pavement level.	1. General excavation to the level of the base-course subgrade.
2. Trench excavation from the top-of-pavement level to the necessary depth for placing the pipe.	2. Trench excavation by hand since the use of the "backhoe" or similar tool is not feasible with the step-in level at the trench, partly at original grade and partly at general excavation level.
3. Pipe placed and trench backfilled with filter material to the top-of-pavement level so that the excavation for the necessary thickness of pavement and the base course will not seal up the filter material.	3. Pipe placed and the remainder of the work completed as shown in Fig. 8(*b*).

In the first procedure, trench excavation volume increases with consequent reduction in general excavation and loss of filter material which is first placed and then removed. The second procedure specifies much slower operation of trench excavation by hand, which in normal progress schedules would be a bottleneck. A solution of all these difficulties is shown in Fig. 37. This

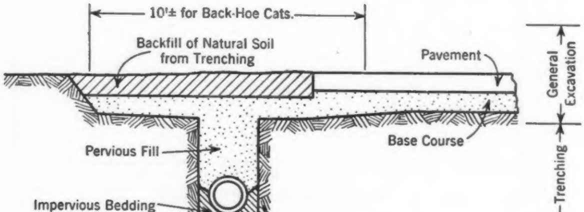

FIG. 37.—BASE-COURSE DRAINAGE DETAIL TO PERMIT MACHINE EXCAVATION

method permits excavation by machine and at the same time introduces an extension of the filter material to help eliminate the sealing up of the filter material by the adjacent soil during heavy rains or from subsurface drainage flow.

The foregoing comments agree with Colonel Stratton's statement in the "Summary" that "Good designs may be vitiated by poor construction * * *," but good designs must also take account of construction methods, tools, and labor available for the work.

ROBERT HORONJEFF,[14] JUN. AM. SOC. C. E.[14a]—As stated by the author, the design of concrete pavements is generally based on the formulas developed by Dean Westergaard. The modulus of soil reaction "*k*" is measured in the field. With a known *k*-value and an assumed working stress for concrete, the required thickness of a slab for any desired wheel load can be computed by

[14] Asst. Chf., Civilian Engr., U. S. Engrs., War Dept., San Francisco, Calif.

[14a] Received by the Secretary April 24, 1944.

substitution in the Westergaard formula. The question in the writer's mind is whether the Westergaard analysis is adequate for determining thicknesses of concrete pavements laid on marshy ground. The load deformation curve (Fig. 4) of a plate 30 in. in diameter placed on a subgrade underlain by mud varying in depth from 40 to 60 ft is considerably different from the curve shown in Fig. 4. When the subgrade consists of mud covered by select material not exceeding 2 ft in depth the load-deformation curve is very flat, indicating that very little load is required to deflect the slab a large amount. In fact, in many cases it is not possible to load the plate with a total of 60,000 lb (Class I airports) without pushing the slab into the mud. With average subgrade conditions the general practice has been to limit the size of individual slabs (by means of dummy joints) to an average dimension of 12.5 ft by 15 ft. The argument presented is that by limiting the size of the individual slab the stresses produced by shrinkage and warping are reduced to a minimum, thus reducing the formation of cracks.

It is the writer's opinion that this argument cannot be applied to construction of pavements on marshy ground. The individual slab sizes should be increased so that a greater bearing area is provided. It would take a larger load to push a 25-ft by 25-ft square slab into the mud than it would to push a 12.5-ft by 15-ft slab. The next questions that arise are how one can be assured that the larger slab will distribute the concentrated wheel load and how the tensile stresses produced by warping will be taken care of. These conditions can be met by proper distribution of reinforcing steel through the slab. At the top the reinforcing steel should be concentrated at the corners and at the edges and at the bottom toward the center of the slab.

The method of computing the size of reinforcing steel is still an unknown factor. It is true that reinforcing steel may not prevent initial cracking of the slab but it will prevent the enlargement of these cracks and it will certainly prevent the complete severance of a portion of the slab from the parent slab, such as is the case with a plain concrete slab. Reinforcing steel has a definite place in the design of concrete pavements on marshy ground.

The transfer of wheel loads of from 60,000 lb to 75,000 lb by means of ordinary steel dowels is somewhat questionable. There is not sufficient surface area in the dowels to transmit the load to the concrete without crushing the concrete. There is a due need for a satisfactory load transfer device that is practicable to install in the field.

Another problem that is constantly arising in airport work is the strengthening of existing concrete pavements by "second-storying." "Second-storying" means the placing of a new slab over an existing slab and separating the two by means of cushion course. The thickness of cushion course will vary depending upon the particular conditions at the airfield involved. Thus, if the strengthening is required around existing hangars, the finished grade of the new slab has to be maintained as close as possible to the grade of the existing hangar floors, which results in the use of a minimum cushion course of from 1 in. to 2 in. between the two slabs. On the other hand, on a runway the grade of the new slab is not limited by existing structures and the cushion course can be as thick as 2 ft.

A study to determine the behavior of slabs placed on existing slabs with various thickness of cushion course and subjected to moving wheel loads up to 75,000 lb has been undertaken by the San Francisco District in conjunction with the staff of the Engineering Materials Laboratory of the University of California, at Berkeley. The aim is to determine the stresses in the slab at such critical points as those on the edges and corners by measuring internal strains in the concrete. The subgrade reactions and surface deflections are also to be measured. From this study it is hoped that some information will be obtained as to the effect of the thickness of cushion course on the structural behavior of the slab. Various thicknesses of slab both reinforced and unreinforced will be studied in the test section.

Another point that usually is given little consideration in the design of airfields is the radius of the fillets applied to runways and taxiways. Present standards are inadequate to a certain extent. These standards set up by the Civil Aeronautics Authority are as follows:

Angle of intersection	Radius of fillet (ft)
0°–85°	25
85°–115°	50
>115°	100

At a number of airfields designed by the writer the using service has arbitrarily increased the radii of the fillets because most of the planes were "cutting the corners." A number of people attribute this condition to "pure carelessness on the part of the pilots," but it is a known fact that, at a number of airfields used both by large transports and single-seat pursuit aircraft, a sincere effort has been made by the pilots to "stay on the pavement." For large transports the difficulty seems to arise from the fact that the ratio of the wheel spread to the paved width of the taxiway is quite large. Therefore, when a pilot is confronted with the need for making a turn on the taxiway the plane veers off from the center line of the taxiway just enough to throw one of the wheels off the pavement. Pursuit aircraft seem always to taxi at fairly rapid speeds and, therefore, in order to maneuver sharp turns the pilots run off the pavement. The writer recommends that the minimum radius of fillets for airfields to be used by fairly large planes be established at 75 ft.

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(24) **"Surface Runoff Determination from Rainfall Without Using Coefficients,"** by W. W. Horner and S. W. Jens, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 1039. (*a*) Fig. 5(*a*), p. 1052.

(35) **"Modern Airfield Planning and Concealment,"** by Merrill E. De Longe, Pitman Publishing Corp., New York and Chicago, 1943.

(36) **"Some Experiments on Shallow Flows Over a Grassed Slope,"** by W. O. Ree, *Transactions*, Am. Geophysical Union, 1939, p. 653.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

SURGES IN PANAMA CANAL REPRODUCED IN MODEL

Discussion

BY HARRY LEYPOLDT

HARRY LEYPOLDT,[7] ESQ.[7a]—The data shown in Figs. 3 and 4 lend themselves to a far different interpretation than the one advanced by the authors, because the water-surface profiles show that a seiche with a period of about 29 min is present in the waterway. Let y = 41.0 ft; $V = \sqrt{g\,y}$ = 36.4 ft per sec; and

$$t = \frac{2\,L}{\sqrt{g\,y}} \qquad (2)$$

in which t is the time in seconds (29 × 60). Computing the uni-nodal length of basin that will accommodate a seiche with a 29-min period, these values in Eq. 1 give L = 31,668 ft. This brings the end of the basin at about station 1614, which is obviously not a place where wave reflection can occur. From this consideration, at least a bi-nodal seiche is necessary to provide the oscillations shown. Assuming the same depth in the dredged cuts leading to the lower reaches of Gatun Lake, the loop end for a bi-nodal seiche would be at the head of the bend in the channel.

That this is the correct interpretation can be established. The gaging stations between the node at station 1772 and the loop at the locks, station 1930, should show high water when all the other gaging stations show low water. This is the condition shown in the profiles. The location of a gaging station at station 1478, very close to the node at station 1456, should result in water-surface profiles with the amplitude of the seiche nearly zero. This is certainly the case in every profile for station 1478. Also, the profiles at station 1550 and station 1656, lying on either side of the center loop, should be nearly similar, which they are. Furthermore (see the profiles from Test No. III) where a fresh impulse is given at an 88-min interval, the amplitudes should be greatest

NOTE.—This paper by F. W. Edwards and Edward Soucek was published in January, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1944, by Harold A. Weggel.

[7] Los Angeles Harbor Dept., San Pedro, Calif.

[7a] Received by the Secretary April 7, 1944.

since 88 is 3 × 29 +; that is, the seiches from the two impulses would augment, whereas those in Test No. V (Fig. 4(*b*)), at the 44-min interval, should be least, since the seiches would interfere to the maximum extent possible. This is likewise the case on the given profiles. No reason was advanced for this choice of intervals.

Without further data, it is conceded that the system may have additional nodes. This can be verified easily on the prototype by gaging stations in the southern part of Gatun Lake. However, treating the problem as a bi-nodal seiche is valid for this discussion. There are minor seiches riding the major one of the prototype, which were probably produced in the canal by reflecting surfaces at places where the width or direction of the canal changes. They do not affect the problem.

The water-surface profiles of Fig. 2 do not show the actual conditions. They are seriously influenced by the findings of Professor King, who states definitely that[3] the water is moving in an open canal when water is added or removed. In the prototype, as in all other seiches, the wave form is nearly sinusoidal, instead of the odd assortment shown in Fig. 2. It is also irrelevant to cite the work of Colonel Brown[4] since his paper treats of tidal canals.

The position of Gatun Lake proper does not influence the part of the canal under consideration in the same manner that lakes do in Colonel Brown's studies. About 6 miles of narrow, shallow waterway, except for the dredged cut, intervenes.

The nomenclature adopted by students of seiche phenomena limits the term "surge" to the "to and fro" motion of the water (current), whereas the rise and fall is called "seiche," and corresponds to tides in tidal waters.

Seiches in nature are produced chiefly by differences in barometric pressure on water surfaces in the same oscillating area, or by tidal effects. In canals, as in the case under discussion, the change in pressure is effected by release of water into the locks.

Since the model was built to scale, and the seiche period is a function only of length and depth of the oscillating system, the model should be a good replica of the prototype. It would be astonishing if it were not; so no conclusions regarding model operations are valid from these experiments. Width of channel does not enter the problem, and therefore added breadth in the model will not change the period, provided depth is held constant. This may be of value in future experiments.

Roughness variations between model and prototype would only cause a difference in the number of oscillations before friction consumed the energy.

The primary object of the investigation was to study the effect on ship operation. In Los Angeles Harbor, numerous continuous seiches occur. Those with relatively long periods have little or no effect on ship maneuvering, whereas the short period seiches are a serious problem, especially in the naval base area where caisson gates for graving docks must be fitted into rather small keyways during the closing operation. The surges accompanying the seiches,

[3] "Translatory Waves in Open Channels," by Horace W. King, *Civil Engineering*, June, 1933, pp. 319–321.

[4] "Flow of Water in Tidal Canals," by Earl I. Brown, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), pp. 749–834.

with their rapid direction reversal, make the locking operation difficult. In the Panama Canal case, additional locks, with shorter filling times or greater water removal, would induce the same period seiches, but with greater amplitude, and therefore with increased surge. This is shown by the small amplitudes in Test No. 1 where slow removal occurs. However, two sets of locks, with filling operations regulated to time periods that would produce interfering seiches, should answer the problem. Another solution would be to have the filling water removed from the main canal into a paralleling conduit of such length (to be determined by model experiments) and at such places as to produce the least seiches due to interference phenomena, the water having sufficient capacity to fill the locks. The locks would be filled from the auxiliary conduit, the seiches would be reduced through interference, and little or no surge would be experienced when the lock gates were opened.

From the nature of the factors governing the period of the seiches (depth and length of oscillating basin), it is apparent that changing the depth or the length would change the period. The depth enters to the one-half power, requiring greater change to produce the same effect that a change in length would produce. Such change in length of oscillating basin could be attained by introducing a gate in the waterway at a suitable place, the gate to remain closed during lock filling.

However, in view of the fact that the surges (currents) are theoretically zero at the loop ends and greatest at the nodes, the existence of a loop at or near the locks would signify that very little current occurs here and therefore that no cause for anxiety is apparent.

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DISCUSSIONS

SEDIMENTATION IN RESERVOIRS

Discussion

By Berard J. Witzig

Berard J. Witzig,[14] Assoc. M. Am. Soc. C. E.[14a]—The able engineers who discussed the paper have done much to clarify several aspects of the problem of sedimentation in reservoirs.

Messrs. Johnson and Stanley stress the confusion that exists in the lack of proper definitions and terms for material moving by the various modes of transportation. To eliminate this ambiguity in the paper, and to avoid confusion in the future, the writer suggests the following definitions:

(*a*) "Wash load" is the relatively fine material in near permanent suspension, which is transported entirely through a stream system without deposition.

(*b*) "Suspended load" is the coarser suspended material, derived from the stream bed, during changes in magnitude of the stream discharge.

(*c*) "Bed load" is the material derived from the stream bed, varying in size from silt to boulders, moved along the stream bed by rolling, sliding, or saltation.

(*d*) "Bed material" is the source of both the coarse suspended load and the bed load, and consists of the geologic formations and alluvial deposits through which the stream channel is cut.

The wash load, the greater part of which may be carried by the rising stages of the flood flow, bears no direct relation to discharge because of the seasonal variability of supply, affected by erosion, meteorologic, and hydrologic conditions on the watershed. However, because the coarse suspended load and the bed load are derived from an always available supply, relations between such loads and the water discharge exist, but may be different in different streams, or at different points on the same stream. Most total suspended-load measurements include the wash load. Hence, the lack of a relationship between the

Note.—This paper by Berard J. Witzig was published in June, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1943, by Joe W. Johnson; October, 1943, by John W. Stanley, Stafford C. Happ, and Thomas H. Means; November, 1943, by Carl B. Brown, and C. S. Jarvis; December, 1943, by Hugh Stevens Bell, and Harry F. Blaney; March, 1944, by A. L. Sonderegger; and April, 1944, by L. Standish Hall.

[14] Engr. (Civ.), U. S. Engr. Office, Buffalo, N. Y.

[14a] Received by the Secretary April 19, 1944.

sediment loads so measured and the water discharge may mean only that the wash load constituted a preponderantly influential part of the total suspended load. Such measurements might be corrected, if the mechanical analyses of the sediment were known, and the boundary range of particle sizes were determined between the materials carried by the two modes of transportation.

No rigid natural distinction actually exists between the coarse material in suspension and the bed load. An arbitrary distinction is necessitated only by present limited knowledge of the phenomena of sediment transportation, and each condition must, therefore, be measured and evaluated by different methods. The writer believes that the so-called "dividing grain size" is only apparent, and may logically be expected to vary with change in stage or discharge at a given point on a stream, as well as with a change in location on the same stream or a shift to a different stream.

Mr. Stanley correctly interprets the writer's implication that no net settling occurs during sediment transportation in suspension. When the suspending forces are sufficiently strong, there is no accumulative deposition, but, because of the very nature of the modes of transportation, there must be an interchange of material between the load in suspension and that in movement or stationary on the bed.

Messrs. Johnson and Means add data on the so-called "silt rating curves." Their cautions in the use of such curves should be heeded, since these relations are not "rating curves" in the same sense as are stage-discharge relations above a real hydraulic control at a gaging station. They are inherently subject to the errors of including wash load, and to the approximations and errors unavoidable in measurement of suspended sediment. Moreover, they cannot include, intrinsically, any factor for bed load.

The writer believes that laboratory analysis of suspended-load samples could be refined to distinguish between wash load and suspended bed-material load, and, therefore, a relatively more accurate estimate could be made of the relation between the temporary suspended load and the water discharge. The breakdown might be made by comparison of suspended load samples with fresh bottom deposits sampled immediately after flood flows. However, there is a possibility that the wash load, varying independently of the discharge, may act either as a flux or, conversely, as a retardant; on the carrying capacity of a given discharge for the bed-material load. Only experimentation could settle this question.

With reference to the use of the silt rating curves, the writer reiterates his opinion that such relations are best interpreted not as actual physical relations but rather as statistical correlations, which may be validly used in estimating the probable average suspended loads carried by given water discharges. Perhaps doubt as to their validity for this purpose would be lessened if the envelopes enclosing the measurements were also drawn, so that it could be seen at a glance that the selected curve represents the probable average in the range of probable concentrations. Thus, the silt rating curve cannot be used to state the sediment concentration in a flood for which no sediment measurements were made, without probability of considerable error, any more than a flood-frequency curve can be used to state that a flood of a given magni-

tude will occur in a certain period. Both are statistical relations and should be used only as such. They can be employed to determine average annual occurrences. Whether the silt rating curve is used with a discharge-duration curve, as suggested by Professor Johnson, or with a flood-frequency curve, as in the writer's example, is immaterial, and would depend only on the extent of data available. Professor Johnson's method would require a longer period of silt sampling than the writer's; the data on the Cuyahoga River in Ohio cover too short a period to enable a comparison to be made.

Mr. Means properly warns against the indiscriminating use of "general relations" or formulas. The writer does not suggest that any of the formulas or procedures be so used. There is no adequate substitute for carefully observed data, but frequently, in their absence, or because time and expense prevent more thorough investigations, the short cuts provided by general formulas may give results of sufficient accuracy. Mr. Means questions Eqs. 2 and 3. Although these equations are intended to express what may prove to be a general law, reference to the source papers (14)(15)(21)[14b] shows that appreciable deviations exist. If this is admitted, proper use of these and similar equations can be made as engineering tools.

Professor Johnson clarifies the (ϕ-ψ)-relationship for bed load, developed by Mr. Einstein (21). The relationship seems based on and verified by a large mass of observed data. Equal substantiation of other methods of evaluating bed load is not known to the writer. However, the (ϕ-ψ)-relationship also requires evaluation of "a certain representative grain diameter." Because of this necessity, and the fact that many observations have varied considerably from Mr. Einstein's adopted curve, quite as much error may result in its use as in application of the simpler Schoklitsch bed-load formula.

Mr. Stanley carefully expounds his reasons for questioning the data on silt settling rates in Table 2, which was intended principally to illustrate indirectly the magnitude of forces required to keep sediment grains of different sizes in suspension. The additional data in Table 6 are of interest. These data illustrate the variation from theory of results obtained by different workers, which probably is due to laboratory procedure and control, material observed, and personal error in observation or interpretation, as well as some imperfection in the basic theory or formula. Because of the great variety in nature, such differences always might properly be expected.

Mr. Stanley suggests that the determination of reservoir silting "indexes" be further refined by consideration of the average annual rainfall and the average susceptibility to erosion. Evaluation of these factors would undoubtedly aid in presenting a more clear-cut delineation of the indexes, inasmuch as those derived in the paper are based on large geographical regions having considerable variation in rainfall and erosion characteristics. The writer has not had the opportunity to carry the study to this extent, however, because of lack of data.

A relation between rainfall and suspended load, on small watersheds, has been demonstrated by Professor Johnson (65). Mass curves of the deviation

[14b] Numerals in parentheses, thus: (14), refer to corresponding items in the Bibliography (see Appendix II in the paper), and at the end of discussion in this issue.

of suspended load from the mean closely parallel the mass curves of deviation of rainfall. The correlation obtainable therefrom may permit computation of suspended load from rainfall records for a period for which suspended load measurements are not available.

The writer does not know of any method of evaluating the average annual erosion from a watershed, except by surveys. As Mr. Brown states, the average annual erosion differs, depending on whether erosion is considered as the eroded material actually delivered to a reservoir or the material eroded from its source and deposited elsewhere on the watershed (colluvial and alluvial deposits). The former would be the factor to be evaluated in the present case. The magnitude of this factor can be stated in terms of annual accumulation of silt per square mile of drainage area, as shown by Mr. Brown in Fig. 6.

Mr. Happ clarifies the findings of the SCS relative to the influence of sheet and bank erosion. The writer was evidently in error in contrasting Mr. Horton's statement with the SCS conclusions. Identification of the principal source of erosion is of vital importance in attacking the problem. However, stream bank erosion is the principal culprit not only in the semiarid Southwest, but also in some humid basins in other parts of the United States. Thus, an SCS survey (52*a*) of the Buffalo Creek, New York, watershed, an area of 437 sq miles, indicates that stream bank erosion produces 80%, and land surface erosion, 20%, of the total sediment reaching lower Buffalo River. Buffalo Creek watershed can be called neither mountainous nor forested, although the headwaters are in the foothills of the Alleghenies. The annual rainfall on the watershed averages about 40 in., and is fairly well distributed throughout the year. A similar situation appears to hold for the Cuyahoga River. Although no erosion surveys of this watershed have been made, the writer's observations indicate that most of the eroded material carried to the lower Cuyahoga at Cleveland is produced from the stream banks and gullies in the lower half of the watershed. The river flats, the upland areas, and most of the headwaters area are of gentle slope, largely unwooded, and generally unaffected by sheet erosion.

Mr. Means pertinently draws attention to the important volume occupied by deposits above reservoir level. This was not emphasized sufficiently by the writer. Such deposits may be of serious consequence, if they should be eroded and again deposited in the reservoir cavity, or if the dam is later raised when they would detract from the added storage that might otherwise have been obtained. Use of vegetation screens, debris barriers, or desilting basins above the reservoir proper should therefore be planned with such future developments in mind. In this connection, Mr. Hall suggests several considerations for the design of debris barriers and vegetation screens. In addition, however, the designer should not disregard the possibility that resultant aggrading of channels and flattening of stream slopes upstream from such works may increase flood damages above the reservoir. If this damage cannot be prevented or avoided, then the damage, or its mitigation, should properly be included as a cost of the reservoir.

Mr. Brown epitomizes the problem when he states that "Control of silting begins when the dam design takes shape on the drawing board." Except for

physical destruction of the dam itself, silting is the only force that can destroy the reservoir; the dam can be replaced, but loss of reservoir storage through silting has no completely efficient economical remedy. Mr. Brown properly questions the erroneous generalities that can be drawn from Fig. 3 and Eq. 6. Fig. 3, from which Eq. 6 was derived, represents an apparent correlation between rate of silting and reservoir capacity, and summarizes data that would otherwise be merely a mass of statistics. As evident from the plotted points, many of the smaller reservoirs have higher rates of sedimentation, when reduced to a common unit basis, than some of the larger. It was not intended to imply the existence of any natural law that the silting rate increases indefinitely with the size of the reservoir, but merely that, if a reservoir of a certain capacity per unit of drainage area were built, it might be expected to silt up at a rate similar to that of an existing reservoir in the same region. There must be a maximum limiting silting rate, regardless of the size of the reservoir, just as there is a limit to the rate of runoff from a watershed, regardless of the reservoir size that may be provided to store that runoff. If a reservoir of sufficient size to store the greater part of most floods from its drainage area silts at a certain rate, there probably would be but little increase in the rate of silting if greater storage had been provided at the same site.

Mr. Brown contributes a valuable discussion on sediment trap efficiencies. His distinction between net and gross erosion is important and should be kept in mind. Eq. 12 defines the trap efficiency coefficient in terms of silting and erosion rates only, which is simple and correct for a particular reservoir. Eq. 7, however, attempts to include the effect of reservoir size, so that known data on existing reservoirs may be applied to proposed reservoirs of different sizes. However, Eqs. 14 and 15, and Fig. 5, present the reservoir designer with a valuable tool for this purpose, and the writer especially appreciates Mr. Brown's contribution at this time.

Fig. 6 illustrates the relation between silting rates and drainage area. It seems reasonable to expect that, with an increase in drainage area, the unit silting rates on large drainage areas should decrease and become constant, since it would be unlikely that all parts of a large watershed would be eroding at the more intense rates found on certain of the small tributary areas. Use of Fig. 6, or of Fig. 3, should be based on thorough consideration of characteristics of and conditions on a basin. Neighboring watersheds often differ considerably; for example, Cayuga, Buffalo, and Cazenovia creeks in western New York, all join to flow into Buffalo Harbor. The three streams drain approximately equal areas, but Cayuga Creek with somewhat lower gradients contributes only about one tenth of the sediment deposited in lower Buffalo River, whereas the other two streams contribute the remainder.

In view of Mr. Brown's statement that SCS surveys have been made "for the purpose of obtaining a representative sampling of silting conditions * * *," the writer retracts his statement "that reservoirs surveyed thus far are more typical of severe rather than of average conditions." The latter statement was based on failure to find a specific utterance similar to Mr. Brown's in available reports of the SCS. Knowledge that SCS silting data constitute representative

samples permits their study to proceed with some assurance that any results obtained are not influenced unduly by extreme conditions.

Mr. Brown has offered a valuable service to the profession in his paper (52) on the control of reservoir silting. It would be extraneous here to enlarge on control measures, and the writer seconds Mr. Blaney's suggestion that those interested review that publication.

Mr. Jarvis calls attention to some benefits resulting or obtainable from sedimentation in reservoirs and canals. However, some sediment deposits do not possess fertilizing value, especially those derived from bank erosion in areas of thin topsoil.

Mr. Bell objects to a misinterpretation of his paper (25) on density currents. Considering that both the original statement and the examples given by Mr. Bell in his discussion are only estimates based on assumed densities of silt in place, the magnitude of the quantities of material that apparently are carried through reservoirs by density currents is thereby illustrated. Whether density currents carry 24% or 35% to 40% of all sediment in Lake Mead, such currents evidently have considerable effect on the rate of sedimentation and may constitute an important factor in its control.

The writer does not agree entirely with Mr. Bell's statement that density currents decrease the specific density of the coarser deposits by removing the finer materials that would otherwise partly fill the interstices of sediments left behind. Density currents carry only the finer silts and clays. Curves given by E. W. Lane, M. Am. Soc. C. E., and Victor A. Koelzer, Jun. Am. Soc. C. E. (66*a*), indicate that the density of material deposited under water increases with an increase in the percentage of coarse material. Control of density flows for voiding silt from a reservoir before deposition might therefore result in final deposits of greater density and less volume. The deposits would also consolidate more rapidly because of the smaller silt or clay content.

Mr. Blaney emphasizes the importance of knowing the relation between silt volume and weight of deposited sediment. Messrs. Lane and Koelzer (66) have compiled all the available data on this subject, and have presented a procedure for estimating the density of deposits at any time after deposition, for various methods of reservoir operation.

Mr. Sonderegger distinguishes between the natural norm of erosion and that caused by man's activities, and states that erosion control can scarcely be expected to reduce sediment production to less than the undisturbed natural rate. The writer does not fully concur that nature cannot economically be improved on at times. When natural rates of erosion are due to the same causes as those accelerated by man, it does not seem an impossibility to modify them by proper remedial measures. Thus, burned-over areas can be reforested; eroding stream banks can be stabilized; and steep gully and stream slopes can be made more gentle by construction of dams and drop structures. The writer agrees, however, that sediment production, and transportation to reservoirs or navigable waters, cannot be prevented entirely at costs commensurate with the benefits.

Mr. Sonderegger properly emphasizes the necessity of providing adequate reservoir storage for silt loss to preserve the hydraulic functions of the reservoir.

In many cases, it might be more economical to build a large dam immediately, rather than to raise the dam shortly after construction, but the decision to do so would necessarily rest on the engineering, financial, and natural conditions peculiar to a given project.

Mr. Hall discusses several special aspects of the erosion problem, and adds data on sedimentation rates in several reservoirs in California, as well as an example of the action of density currents in Pardee Reservoir. His comments on the effects of stream slopes seem to apply essentially to the reworking of flood plain and channel deposits. However, flat stream gradients do not necessarily imply that sheet erosion on the watershed outweighs bank erosion, for many streams of gentle slope flow through alluvial strata of low resistance to eroding forces, although the land surfaces themselves are amply protected by surface cover. A notable example is the Mississippi River below Cairo, Ill.

An interesting comparison is made between the Kennedy and the Schoklitsch formulas, with reference to the velocity and drag required, respectively, to maintain movement of sediment in suspension and along the bed. Although both these formulas are empirical in origin, the comparison would indicate that they are not unsound theoretically.

The extremely small concentration of sediment required to produce density flows is illustrated by Mr. Hall's description of the June, 1935, occurrence in Pardee Reservoir. This example also seems to demonstrate conclusively that density currents carrying silt can be controlled by proper manipulation of outlets. Neither laboratory research, nor large-scale experimentation with existing reservoirs, should be neglected until this presently speculative aspect of reservoir action is usefully applied.

With particular reference to Pardee Reservoir, the writer suggests that the current may have been of such density that it flowed along the top of a cold water layer of just slightly greater density. When the sluice-gates at the bottom of the dam were opened, the resulting hydraulic current drew the silty water to the outlet; when the gates were closed, the hydraulic current over the spillway drew the silty water to the top of the dam. Without further data, the writer would suggest that the concentration of silt in the spillway flow was somewhat less than that through the gates, inasmuch as the density current probably had to mix with a greater proportion of clear reservoir water in rising to the top of the dam, than in sinking to the bottom. Furthermore, the density flow must have been quite thick to have appeared so quickly over the spillway, or a considerable part of the stored water in the reservoir may have consisted of the silty suspension. The shape of the reservoir, and its relation to the location of the dam, as well as the shape of the back surface of the dam itself, may also have influenced the action of the current.

Mr. Hall suggests a method of correlating the rate of reservoir silting with the mean annual runoff from the tributary area. The linear relation illustrated in Fig. 8(*b*) between annual runoff volume per square mile and sedimentation as a percentage of annual runoff seems to fit the plotted data. The writer does not have the necessary information to test this correlation on the source data of Fig. 3. The method is worthy of further investigation, however, and may prove valuable in estimating sedimentation rates from runoff records.

This tool may possibly be applicable to larger watersheds with more accuracy than is the correlation between mass deviation curves of rainfall and suspended load previously mentioned in this discussion. The latter method is unduly influenced by variable distribution of rainfall on large areas.

Eq. 20 is a particular statement of Eq. 11, but is more complete than Eq. 7. However, for practical use, it would require evaluation of a coefficient for the right-hand term. As Mr. Brown suggests in Eq. 12, however, the term E should be the net erosion, E_n, delivered to the reservoir. No practical method is apparent at this time which includes in a single quantitative equation the omitted factors mentioned by Mr. Hall. However, with reference to size gradation of sediment, few particles of the largest sizes are carried to the reservoir except in debris basins as in California, and most particles of the finest sizes are transported through the reservoir, except in the case of very large volumes which store the entire flood runoff for an appreciable period of time. The intermediate sizes, between "Cyclopean" sediment and colloidal silt, generally form the predominant components of reservoir deposits. The flow characteristics of the tributary stream are partly accounted for in the term, Q_a. Reservoir shape and manner of operation would seem to be qualitative factors.

By direct correspondence, C. S. Howard, Chemist of the U. S. Geological Survey, states that the greater part of the annual silt load in a stream may be carried in a relatively short time, with the stream flowing comparatively clear during the remainder of the year. This is illustrated in Fig. 1, where the quantity of 2.76 tons of silt per second was carried by a discharge of 27,800 cu ft per sec. A one-day flow of this magnitude would be equivalent to about 45% of the estimated average annual silt load of 532,000 tons. However, this concentration has about a 1.5% chance of occurrence, or may occur about once in 67 years. Thus, a flood of this magnitude and frequency would contribute over its period of recurrence only about 0.68% of the average annual silt load.

Mr. Howard also points out that the data in the basic paper on the efficiency of the desilting works for the All-American Canal were estimates before operation began. The writer has learned that up to August, 1943, the desilting works had not been put into operation, since it had been possible to obtain a comparatively silt-free diversion to the canal by reason of high reservoir levels and high sluiceway discharges.

The writer has found the discussions very instructive. Practically every topic in the basic paper has been discussed or commented upon, erroneous or doubtful points clarified, and interesting new data and suggestions added. Differences of opinion on theory and interpretation of natural phenomenon, of course, will continue. However, the travail common to the solution of all involved problems will undoubtedly produce valuable social and economic benefits, as the mechanism of sedimentation becomes better understood and practicable remedies are developed for reservoir silting.

In view of the historic fact that water resources are of utmost importance to a civilization and an economy, the present generation should exert every effort to use these resources wisely and to conserve them to the greatest degree for future generations.

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(21) **"Formulas for the Transportation of Bed Load,"** by H. A. Einstein, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), pp. 561–577.

(25) **"Stratified Flow in Reservoirs and Its Use in Prevention of Silting,"** by Hugh Stevens Bell, *Miscellaneous Publication No. 491*, U.S.D.A., September, 1942.

(52) **"The Control of Reservoir Silting,"** by Carl B. Brown, *Miscellaneous Publication No. 521*, U.S.D.A., 1943. (*a*) pp. 116–117.

(65) **"The Use of Mass-Curves in Evaluating Suspended Load Observations on Small Piedmont Streams,"** by J. W. Johnson, *Transactions*, Am. Geophysical Union, 1942, pp. 664–672.

(66) **"Density of Sediments Deposited in Reservoirs,"** by E. W. Lane and Victor A. Koelzer, *Report No. 9* in **"A Study of Methods Used in Measurement and Analysis of Sediment Loads in Streams,"** St. Paul U. S. Engr. Dist. Sub-office, Hydr. Laboratory, Univ. of Iowa, Iowa City, November, 1943. (*a*) Fig. 2, p. 37.

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DISCUSSIONS

STORAGE AND THE UNIT HYDROGRAPH

Discussion

BY E. F. BRATER, L. C. CRAWFORD, ROBERT E. KENNEDY, AND VICTOR H. COCHRANE

E. F. BRATER,[19] JUN. AM. SOC. C. E.[19a]—The unit hydrograph is the most useful tool available for the purpose of converting a quantity of surface water, resulting from precipitation, to a river discharge hydrograph. Since the unit hydrograph concept was introduced, a number of different interpretations and methods of application have been evolved. A description of these various concepts and techniques should be included in technical literature. Such a study and discussions of it should help to unify and improve engineers' understanding of the unit hydrograph. This paper is a valuable step in that direction. However, the engineer who may be desirous of making use of this technique will hope that a more detailed description of the methodology, with examples of the numerical computations, will be included in the author's closing discussion. For instance, the "time-area concentration curve" is an important feature of the method, but the author has not shown how one may be obtained for a given watershed.

In his method of flood routing, Mr. Clark sets up the relationship between storage and discharge in Eqs. 3 and 4. In Eq. 4, x is a term that is allowed to vary from 0 to 1. When $x = 0$, the storage becomes proportional to the outflow from the channel reach; when $x = 1$, the storage must be proportional to the inflow; and, when $x = 0.5$, the storage becomes proportional to both inflow and outflow in equal weight. The third condition assumes trapezoidal or double-wedge storage. Mr. Clark indicates that for most streams x is nearly 0.5. However, he chooses to discuss at considerable length the importance of the case in which x is greater than 0.5. In this connection, the writer does not agree with the author.

In the first place, the case of $x = 1$ is a physical impossibility in a real reach of channel, since, if storage is proportional to inflow, storage must become

NOTE.—This paper by C. O. Clark was published in November, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1944, by James S. Sweet, and Otto H. Meyer; and March, 1944, L. K. Sherman, Gordon R. Williams, and Ray K. Linsley, Jr.

[19] Asst. Prof., Civ. Eng., Univ. of Michigan, Ann Arbor, Mich.

[19a] Received by the Secretary March 31, 1944.

zero at the same instant that inflow becomes zero. For this to be true, the final portions of flood water would have to travel the length of the reach instantaneously. This paradox is illustrated by Fig. 2, in which, for $x = 1$, the inflow hydrograph and the routed or outflow hydrograph end at the same time. Thus, if such a graph were routed through successive reaches, the recession side of the graph could not advance in time. The result of several routings of the author's graph is a number of large plus and minus ordinates occurring at the end of the original hydrograph.

Mr. Clark is of the opinion that storage of this type does occur and that it may cause outflow rates from a given reach to be greater than the inflow rates even without the addition of inflow from the local drainage area. In the third paragraph following Eq. 3 he states:

> "Reduction of the inflow rate [where backwater exists] necessarily would be accompanied by a decrease in storage and an increase of outflow rate. Rapid inflow shutdown necessarily would be accompanied by a release of the backwater storage and a consequent increase in the slack-water storage or in the outflow discharge or in both."

As an illustration of the nature of this action, the author takes the case of uniform discharge in the rectangular channel shown in Fig. 1. He assumes that the uniform flow of 116,000 cu ft per sec is suddenly stopped at the upper end of the reach and concludes that this will cause an increase in the depth at the lower end because 260 acre-ft of water are released from storage at the upper end. The writer wishes to point out that, since there is no change in the conditions at the lower end of the reach, the normal discharge of 116,000 cu ft per sec will continue there until the effect of this sudden stoppage can be transmitted through the length of the channel. A recession type of wave will be started at the instant of shutoff which, neglecting energy losses, might have a velocity as great as $V_w = \sqrt{g\,d} + V = \sqrt{32.2 \times 20} + 11.6 = 37$ ft per sec. Therefore, the earliest possible time that the effect will reach the lower end is $\frac{10{,}000}{37} = 270$ sec after stopping inflow. During this time a volume of $\frac{270 \times 116{,}000}{43{,}560} = 720$ acre-ft will have passed out of the lower end of the reach. This is more than sufficient to make room for the 260 acre-ft which the author wishes to accommodate. There is in no sense a "release" of stored water, due to a decrease in the discharge at the upper end of the reach, either in a channel such as this or in the backwater above an impounding reservoir. Kinetic storage is held in place solely by the resistance to flow from point to point in the channel. It is already in complete equilibrium—that is, flowing as fast as possible with the energy available. A decrease in depth at the upper end of the channel can only decrease the energy and thus retard, rather than accelerate, flow.

L. C. Crawford,[20] Assoc. M. Am. Soc. C. E.[20a]—One phase of this interesting paper pertains to natural channel or valley storage and the relation-

[20] Consultant, Iowa Inst. of Hydr. Research; Dist. Engr., U. S. Geological Survey, Iowa City, Iowa.

[20a] Received by the Secretary April 3, 1944.

ship of this volume to discharge or stage. The channel-storage rating has become a valuable tool in flood routing, hydrograph analysis and synthesis, and infiltration studies. The author's comments on real and relative accuracy are very pertinent to unit hydrograph and channel-storage ratings which may be developed by indirect methods through the analysis of records of discharge during past major rises or capital floods.

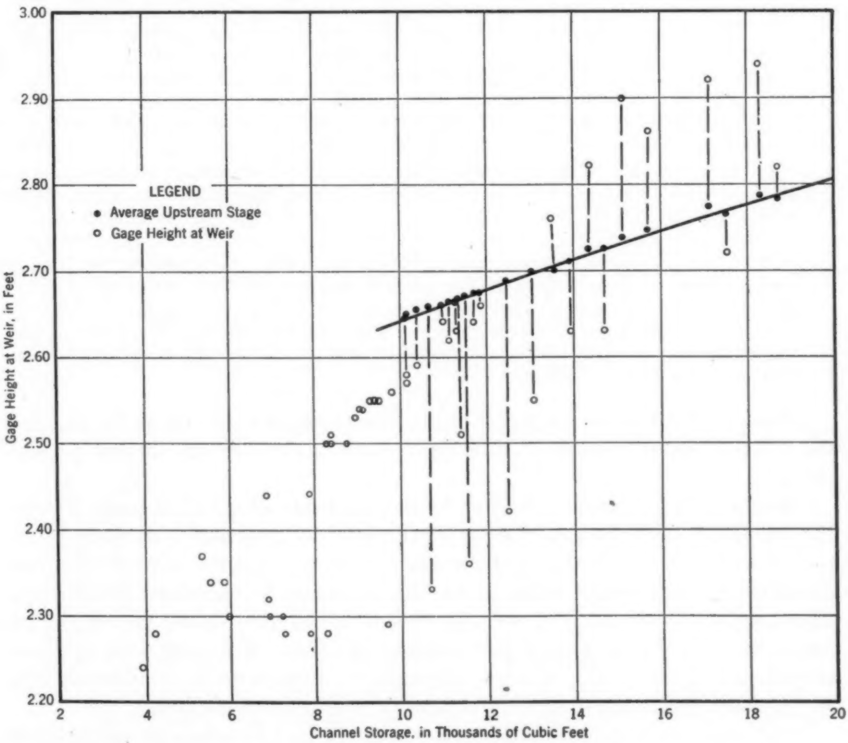


FIG. 13.—CHANNEL-STORAGE RATING CURVE FOR DIFFICULT RUN NEAR FAIRFAX, VA.

The basic stage and discharge records, which are cooperatively collected by the United States Geological Survey and other agencies, give an opportunity for potential channel storage and related studies that probably has not been sufficiently emphasized. Indirect methods of storage determination have been developed and are being further extended through new concepts, such as Mr. Clark presents, and through the use of basic records of stream flow. Thus, it is possible to ascertain with considerable relative accuracy various relations pertaining to natural channel or valley storage that otherwise would be too costly and wholly impracticable without extensive cross-sectional surveys.

Practical experience with recession curves and storage relations have shown, however, that additional refinement in technique may be necessary and de-

sirable. Such refinement is possible if sufficient hydrologic data are collected and correlated with stream flow records.

Fig. 13 shows determinations of gage height and storage in a small head-water area of about 1 sq mile on Difficult Run near Fairfax, Va. The method of channel-storage determinations was presented by O. E. Meinzer and other members of the United States Geological Survey,[21] in the following descriptive discussion:

> "The gaging station is equipped with a V-notch weir and a water-stage recorder. Measurements of channel storage were made by means of 134 secondary gages. The trunk-stream and each of the branches were divided into 100-foot segments, and at the middle of each segment a secondary gage was installed and a profile of the cross section of the channel was made. Each gage consists simply of a 2- by 2-inch stake driven into the bed of the stream so that the depth of water above the top of the stake can be measured with a scale. Secondary gages were installed and cross sections of the channel were made as far upstream on each of the four branches as any stream existed. The volume of water in each segment at any time was computed in cubic feet by multiplying the area of the cross section, in square feet, of the stream at the gaging station for that segment, by 100, and the channel storage in the entire stream system was computed as the sum of the volumes of water in all of the 134 segments. No account was taken, in the computations, of the dead storage below the level of the tops of the 2- by 2-inch stakes."

The observations and calculations of storage were made by V. C. Fishel of the United States Geological Survey in connection with an effluent seepage study.

Despite the probable precision in the determinations of storage volume, the relationship of the gage height at the weir to a storage volume is shown to be somewhat complicated and uncertain. However, if the computed storage is related to an average stage prevailing upstream as calculated from depth measurements from each of the 134 stakes, about two dozen of the storage determinations during appreciable outflow and channel storage may be alined as indicated in Fig. 13. This relationship is interesting in connection with Mr. Clark's discussion of the correlation of storage with discharge.

The measurements on Difficult Run indicate that the stage or discharge at the weir is a very rough index of stages and storage that may prevail in even such an extremely small basin. In addition, it is often presumed that the same relation exists between outflow and storage during rising stages as during the recession. Some methods recognize the existence of this hysteresis effect in connection with channel-storage studies but continue to assume that the net result is small or may be neglected. Some investigators in the field, neverthe-less, recognize the inaccuracies in such an approach.

In any event, the study on Difficult Run demonstrates that more exact analysis is possible and probably desirable when the base data are suitable and sufficient. By relating mean upstream stages with channel-storage determina-tions, evaluations can be made of certain factors whose effect heretofore has been generally given only casual consideration. Moreover, computations in-

[21] "The Channel-Storage Method of Determining Effluent Seepage," by O. E. Meinzer, R. C. Cady, R. M. Leggette, and V. C. Fishel, *Transactions*, Am Geophysical Union, 1936, Pt. 2, pp. 415–416.

volving the volume of channel storage during the rising side of a hydrograph can be made with some increased accuracy from such a storage curve as determined from the recession limb or by other methods.

The use of multiple gages for determining the channel-storage rating also affords a relatively simple method for larger basins. For example, several basins with areas greater than 3,000 sq miles have been examined to explore and demonstrate the procedure presented in this discussion. Fig. 14 shows the

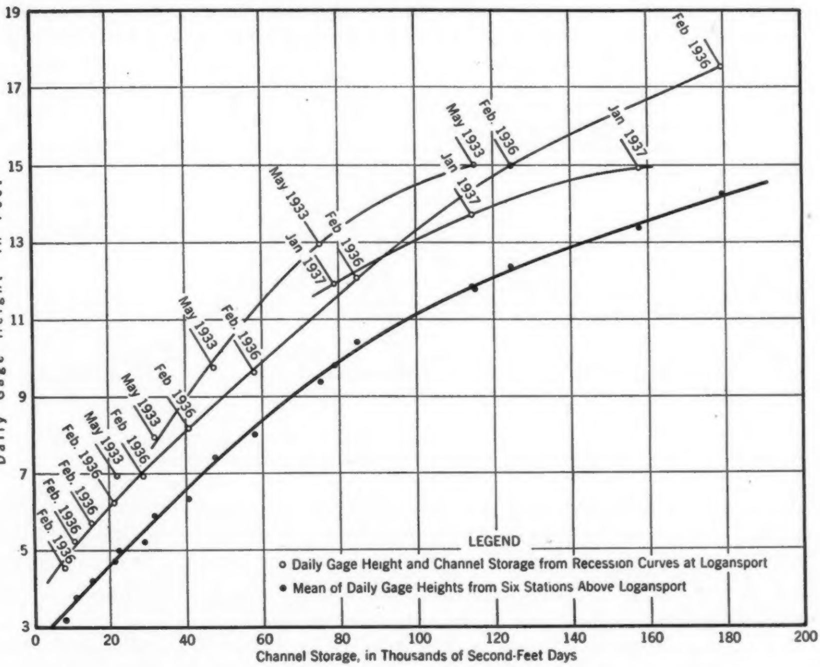


FIG. 14.—CHANNEL-STORAGE RATING CURVE FOR WABASH RIVER AT LOGANSPORT, IND.

orientation of the channel-storage determinations as computed from recession curves for several major rises at the gaging station on the Wabash River at Logansport, Ind. The basin above Logansport comprises 3,760 sq miles and storage takes place in several tributaries with complicated inflow conditions.

Figs. 13 and 14 illustrate important limitations in the assumption of Mr. Clark and others (see heading, "Valley Storage") that "over a large range the true storage may be closely approximated by storage directly proportional to discharge." The method, utilizing an average height or weighted upstream prevailing stage as an index for storage, appears also to provide a somewhat convenient approach for channel-storage determinations during the rising, as well as the falling, stream stages.

The opinions expressed herein are those of the writer and not necessarily those of the organizations with which the writer is identified.

ROBERT E. KENNEDY,[22] M. AM. SOC. C. E.[22a]—At the risk of being more or less academic, the writer would like to comment on the old controversy over the basic assumption of the unit hydrograph.

In Fig. 15 the unit hydrograph, as labeled, is the runoff of a net rain of 1 in. over the watershed in a certain time unit—1 hr, 6 hr, or 12 hr—as may be chosen by the investigator. Then the basic assumption of the unit hydrograph idea is that, when a 2-in. net rain, for instance, falls on the watershed in the same length of time as the 1-in. net rain, the ordinates of the new hydrograph will be twice as high as those of the unit hydrograph, but the time of runoff will be the same for both storms. In Fig. 15 the hydrograph of a 2-in. net rain is

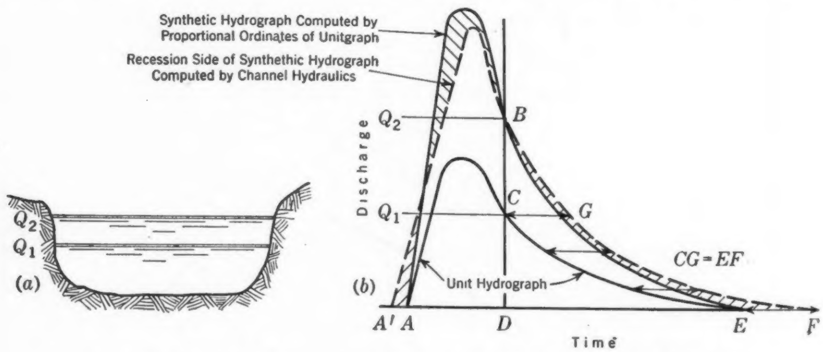


FIG. 15.—COMPARISON OF HYDROGRAPH COMPUTED BY UNITGRAPH METHOD WITH ONE COMPUTED BY CHANNEL HYDRAULICS ON RECESSION SIDE

labeled "Synthetic Hydrograph Computed by Proportional Ordinates of Unitgraph." The time length of both hydrographs, according to the basic definition of the unit hydrograph, is the line AE, for practical purposes. It is not claimed that the base lengths are exactly identical, which opens the question as to how much difference might occur under extreme conditions.

A glance at the cross section of a stream channel in Fig. 15(*a*) shows that the channel storage of the two storms cannot be emptied in the same time. That would mean that the water stored in the channel at Q_2 and represented by point B on the hydrograph in Fig. 15(*b*) must run out in the same time as that stored at Q_1 in the channel and shown as point C on the smaller hydrograph, Fig. 15(*b*). This cannot occur because the water stored in the channel between Q_2 and Q_1 must go first, and that takes time. No matter how fast it may flow, the water cannot escape in "nothing flat"!

Just how much longer time the larger storage requires to empty is susceptible of mathematical treatment. The writer developed this relation and then found that his labor had been largely anticipated in mathematical analyses published by Robert E. Horton, M. Am. Soc. C. E., in 1936[23] and 1937.[24]

[22] Asst. Engr., U. S. Bureau of Reclamation, Denver, Colo.

[22a] Received by the Secretary April 4, 1944.

[23] "Natural Stream Channel-Storage," by Robert E. Horton, *Transactions*, Am. Geophysical Union, 1936, Pt. II, pp. 406–416.

[24] "Natural Stream Channel-Storage" (Second Paper), by Robert E. Horton, *ibid.*, 1937, Pt. II, pp. 440–456.

Any two floods can be so compared on the recession side of their respective hydrographs that the discharge at one point, for example, is twice that of the other. Let these two points be points of contraflexure. Two such stages are shown in Fig. 15(*b*) at point B on one hydrograph (dashed line) and point C on the smaller hydrograph below, ignoring for the present that the latter was used to illustrate the unit hydrograph.

The difference in time required by the channel to empty itself of these two floods from this point of contraflexure is computed as $n^{0.20} - 1$ when n is the ratio of the ordinates of the two stages at this point. In Fig. 15, $n = 2$; and $2^{0.20} - 1 = 0.14$. Lines EF and CG, Fig. 15(*b*), are each 14% of line DE.

When the discharge of the larger flood is three times the smaller one at the point of contraflexure the larger flood requires 25% longer time to empty from that point on the hydrograph. If the larger flood discharge is ten times the smaller flood discharge, the larger flood would take 58% longer to empty. However, in practice the last featheredge of the storm water flow is so diluted with ground water or base flow that the end of the hydrograph is entirely indeterminate.

To complete the dashed-line hydrograph, resort was made to the non-mathematical assumption that the larger storm would take as much longer to fill the channel system as it does to empty it when compared to the time required for the smaller storm, so A′A was made 14% of AD. The rising leg of the dashed-line hydrograph was made parallel to that of the smaller flood.

The two areas shown crosshatched in one direction were made equal to the one area crosshatched in the other direction because the two hydrographs bounding these crosshatched areas represent the runoff from the same storm.

In conclusion it appears from the completed figure that the multiple ordinate concept of the unitgraph is not strictly correct because, when two such hydrographs as shown are superimposed, at only one point is the ordinate of the larger one exactly twice that of the smaller one. No matter how much shifting was done, all the ordinates of the larger or dashed-line hydrograph could not be twice those of the smaller one.

Nevertheless the unit hydrograph is a most useful tool in hydrology.

Victor H. Cochrane,[25] M. Am. Soc. C. E.[25a]—The relationship between the unit hydrograph and the storage capacity of the drainage system, as well as a simple and practical procedure for calculating hydrographs, is presented in this excellent paper. The author rightly states that the determination of a hydrograph depends upon two, large, basic factors—the shape of the watershed and the storage through which the runoff must come. The influence of these two factors is largely accounted for by the use of what the author calls the time-area concentration curve, and both the accuracy and simplicity of the method are due to this device.

Storage is related to time, and may be expressed in time units. The paper contains a discussion of storage concepts, and some rather surprising conclusions are drawn, but it does not appear that the author's procedure is

[25] Cons. Engr., Tulsa, Okla.

[25a] Received by the Secretary April 24, 1944.

dependent upon the validity of these conclusions. If the processes by which a hydrograph is built up are viewed from the standpoint of time instead of storage, a different, and perhaps more easily understood, explanation of the basis of the author's method is derived.

If a drainage basin is divided into a number of areas, or zones, such that their downstream boundaries are at equal time intervals of flow from the point of measurement, a bar diagram of these areas would be the time-area diagram for the watershed, the base of each bar being the constant time interval

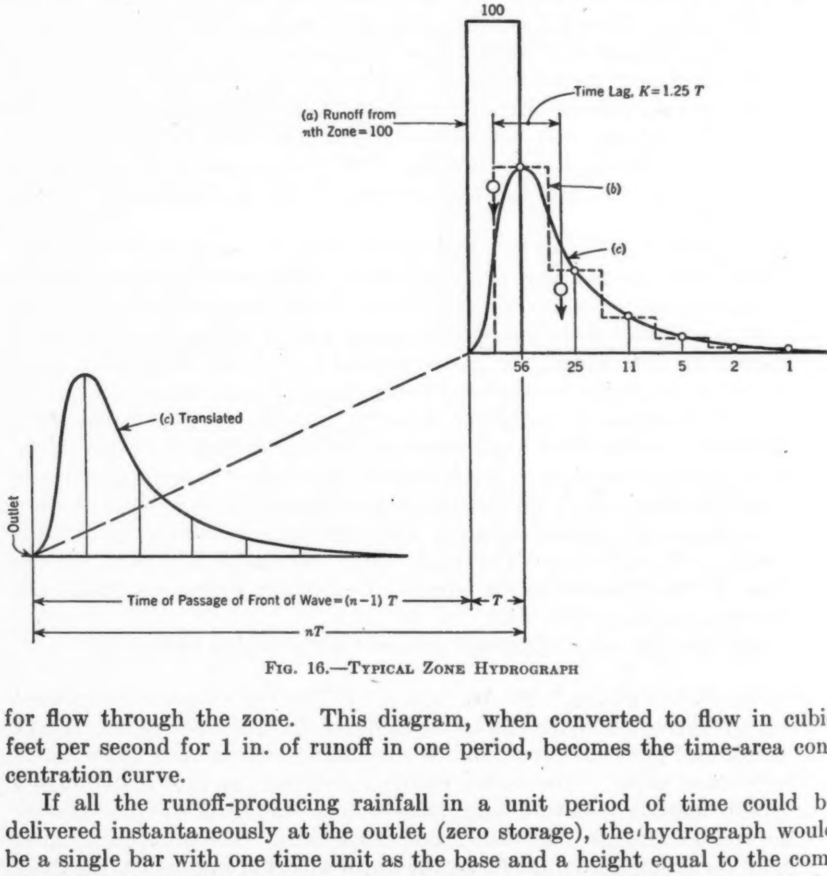


FIG. 16.—TYPICAL ZONE HYDROGRAPH

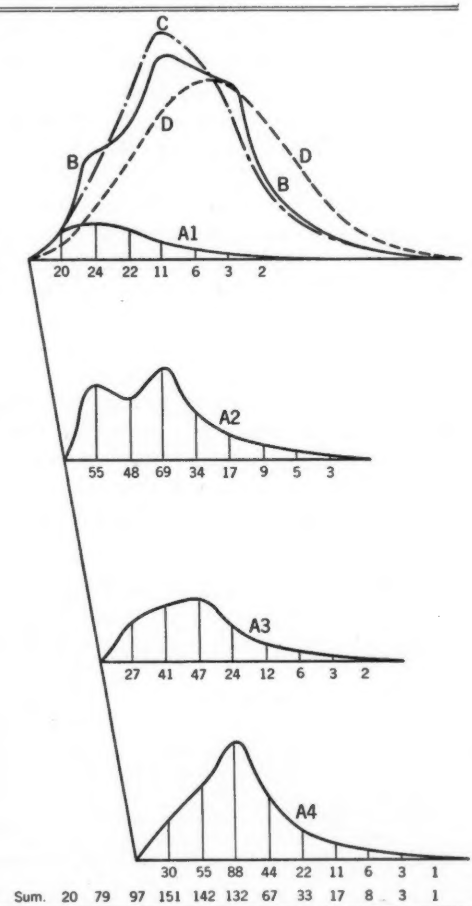
for flow through the zone. This diagram, when converted to flow in cubic feet per second for 1 in. of runoff in one period, becomes the time-area concentration curve.

If all the runoff-producing rainfall in a unit period of time could be delivered instantaneously at the outlet (zero storage), the hydrograph would be a single bar with one time unit as the base and a height equal to the combined height of all the bars in the time-area concentration curve. If the net rainfall in each zone could be concentrated instantaneously at the zone outlet, and if the zone concentrations should then flow to the point of measurement at channel velocity and without further modification, the time-area curve would be the hydrograph of flow at the outlet. This latter process accounts for the greater part of the effect of time (or storage), and the time-area diagram resembles the actual hydrograph.

However, the zone contributions do undergo modification within the limits of the zone and during the passage downstream. In Fig. 16 the bar diagram represents the runoff originating in the nth zone above the outlet during the unit time T. By the time it reaches the outlet it is modified to a typical out-

TABLE 4.—COMPUTATION OF HYDROGRAPH; $K = 1.5\,T$

Zone Number	Area of Zone, in Square Miles	Net Rainfall: Depth, in Inches	Net Rainfall: Miles² Inches	Distribution to Periods: 1	2	3
1	40	2.2	88	40	28	20
2	100	2.4	240	110	40	90
3	60	2.7	162	54	54	54
4	100	2.6	260	60	80	120
Totals	300		750			



flow hydrograph, such as curve (c), which is shown in relation to the precipitation and in the translated position. In this connection the author makes the valuable suggestion that the modification (which is in addition to that implicit in the time-area concentration curve) be determined by means of the Muskingum equations with x equal to zero and with K, the time lag in hours between center of mass of net zone rainfall and center of mass of zone runoff, equal to some small amount, such as 1.25 T. The modified hydrograph be-

comes the bar diagram (b), and if a smooth curve is drawn through the ordinates of (b) it will be the zone hydrograph (c).

The author's use of a constant value for K seems to be based on the assumption that most of the modification occurs within the limits of the zone. This is a reasonable basis for practical use, for it will be found that the precise shape of the zone hydrograph is of little consequence. However, it is not difficult to make use of variable values of K. The summation of the zone hydrographs for a one-period rain may be used as a unit hydrograph, but it is better to complete each zone hydrograph for its entire runoff and then combine the flows.

The hydrograph is sensitive to the shape of the time-area diagram and to the number of periods of rainfall. Variations in rainfall intensity and distribution, infiltration losses, and other factors may be taken into account zone by zone. The method is simple and flexible. It is based on the same conception of constant time elements that underlies the unit hydrograph theory, but it is more adaptable to a variety of conditions. The hydrograph may be computed for an approximate time interval and then corrected to fit the proper time base. When the time-area diagram and other constants are properly determined, the hydrograph due to any type of storm can be calculated with satisfactory limits of approximation. Conversely, any kind of storm can be employed in deriving the constants.

Table 4 shows the suggested arrangement of data. The calculations are for a four-zone watershed having an area of 300 sq miles. The zones vary considerably in size, and a three-period rainfall is irregularly distributed both with respect to zones and time periods. Constant K equals 1.5 T, so that $C_0 + C_1 = C_2 = 0.50$. The modification of runoff for zone 1 is computed as shown in Table 5.

TABLE 5.—COMPUTATION FOR THE MODIFICATION OF RUNOFF, ZONE 1, TABLE 4

Line No.	Description	Runoff in Mile²-Inches							
		40	28	20	..	..	..	..	Remainder
1	Runoff, in mile²-inches × 0.50	20	14	10	..	..	..	..	..
2	Line 3 (advanced one period) × 0.50	..	10	12	..	..	..	..	..
3	Time-period ordinates	20	24	22	11	5.5	2.75	1.38	1.37
4	Recorded in Table 4, and shown graphically	20	24	22	11	6	3	2	..

The zone hydrographs, A1, A2, A3, and A4, are combined by adding the computed ordinates, resulting in the irregular hydrograph B (see Table 4). The relatively small amount of precipitation in zone 2 during the middle time period is the cause of the dip in A2 and the corresponding flattening of B. The sharp peak in A4 results in a bulge in B. If the same total net rainfall were uniform over the entire drainage area over the three time periods, the resulting hydrograph, C, is much less irregular, but the peak is changed only slightly. The same total rainfall uniformly distributed over five periods would produce hydrograph D, having a considerably lower peak.

If the surface and subsurface flows are separated, the foregoing method may be applied in the case of the Appomattox River (see heading, "Derivation of the Instantaneous and Unit Hydrographs"). Assuming a uniformly distributed net rainfall of 2.35 in. (3,140 mile²-inches), occurring in the three periods ending at noon of April 26, and a time lag (storage) of 15 hours, then adding subsurface flows amounting to 0.57 in. in 10 days in accordance with curve (c), Fig. 8, the results agree substantially with the computed hydrograph shown in Fig. 9.

In the "statement of fact" No. 1, Mr. Clark states that there may have been a considerable variation in rainfall from east to west both in quantity and time. The writer assumed a variation of from 1.7 in. to 3.1 in., with a weighted average of 2.35 in. over the entire basin, and found that, on the whole, the computed curve did not fit the observed hydrograph any better than did the author's hydrograph, Fig. 8. The effect of varying K from a minimum of 9 to a maximum of 20 was found to be negligibly small. It would seem that the time-area diagram is the most important factor in the computation. In all cases there was too much difference between the peaks and the valley between, as compared with the recorded flows. It is likely that the record is at fault, as suggested in the "statement of fact" No. 3, or that the largest zone areas have been somewhat overestimated.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

PENDLETON LEVEE FAILURE

Discussion

By Kenneth E. Fields and William L. Wells

Kenneth E. Fields,[14] and William L. Wells,[15] Assoc. Members, Am. Soc. C. E.[15a]—Professors Peck and Terzaghi suggest that the failure may have been conditioned by the presence of a thin horizontal layer of weak material not disclosed by the borings. It is hoped that it will be possible to obtain continuous cores of the clay stratum at the site at some future date when the demands of Mars have been met.

Professor Peck describes a water pressure measuring device used in connection with the Chicago Subway project which seems more simple and easy to install than the Pendleton devices.

Professor Terzaghi is of the opinion that failure occurred along a fairly continuous horizontal layer of sand or silt near the middle of the clay stratum. The writers agree that the failure occurred along a horizontal plane within the clay stratum; but all the data obtained indicate that the plane of failure was near the bottom of that stratum. Pore-water pressure was a maximum and effective pressure a minimum near the bottom of the clay stratum on the day of failure at all piezometer locations except those beyond the embankment toes. The pore-water pressure distribution shown in Fig. 8 is typical for all piezometer locations beneath the embankment on the land side. Pore-water pressures were not equal to or greater than total overburden pressures except in zones immediately beneath and beyond the embankment toes. Thus, with strength a minimum near the bottom (assuming homogeneity) and shearing stress a maximum, it is logical to assume that failure occurred along or near the bottom of the stratum.

Professor Terzaghi bases his argument for the presence of fairly continuous layers of highly permeable material in the central part of the clay stratum upon Fig. 10 which contains hydrostatic pressure distributions at a point 175 ft

Note.—This paper by Kenneth E. Fields and William L. Wells was published in December, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1942, by Ralph B. Peck, Jun. Am. Soc. C. E.; and June, 1943, by K. Terzaghi, T. A. Middlebrooks, and D. P. Krynine.

[14] Lt.-Col., Corps of Engrs., U. S. Army, Fort Benning, Ga.

[15] Capt., Corps of Engrs., U. S. Army, Vicksburg, Miss.

[15a] Received by the Secretary April 14, 1944.

land side of the center line, or about 15 ft beyond the land-side toe of the embankment. This value indicates a hydraulic gradient directed toward the central portion of the stratum. Such a distribution of pore-water pressures was exceptional rather than general within the land-side portion of the foundation. Further, at and beyond the toes of the structure, flow would be directed away from the silt or sand partings as illustrated by Fig. 18(*b*) of Professor Terzaghi's discussion, should such partings be continuous beneath the embankment.

Professor Terzaghi then states that the excess pore-water pressures in the clay beyond the toes of the fill can be explained adequately by the sedimentary origin of the clay—that is, by the fact that the permeability of the clay in a horizontal direction is much greater than that in a vertical direction. In the original report[16] on the subject the following three theories were considered as possible explanations of the pore-water pressure distributions observed:

(*a*) Arching in the fill resulting in a spreading of the load laterally;
(*b*) A fluid-like transmission of pressure laterally in the clay stratum;
(*c*) Lateral drainage away from the zone of maximum pressure.

All these theories were rejected as primary causes of the pore-water pressure distributions noted since they do not explain the large variations of the hydrostatic excess pressure with depth (Fig. 8) and the only theory which apparently explains the phenomenon adequately is that a transference of intergranular pressure to hydrostatic or pore-water pressure occurred within zones of maximum shearing stresses. This phenomenon was called "remolding" by the writers, a term which may be too extreme, as suggested by Professor Peck, since it may occur at relatively small strains. In the Pendleton case, the strain within the clay may not have been so small since the base of the embankment near its center settled a total of about 1 ft up to February 12, 1940, two days prior to failure, and one settlement plate 150 ft land side, near the toe of the embankment, indicated an apparent uplift of about 1 ft as early as January 12.

The abrupt rise in the readings of all piezometers located between points 50 ft river side and 175 ft land side, which commenced one day before failure and continued for one to two days thereafter, appears to be further evidence that, prior to and during failure, a transference of intergranular pressure to pore-water pressure occurred.

Professor Terzaghi inquires as to the type of bentonite used for sealing the drill holes used for the piezometers. The bentonite used was a standard commercial grade having a grain size of 0.3 to 0.5 mm. This material exerts some pressure when confined and supplied with water. In the present case, however, the situation is somewhat different since any water which the bentonite absorbed must be supplied by the pore water within the clay. Six of the piezometers installed were located 225 ft from the center line of the embankment, or about 65 ft beyond the toes. Three of these piezometers, on the river side, did not indicate hydrostatic pressure excesses at any time. The other three, on the land side, indicated only normal hydrostatic pressure up to January 24, at which time they had been in place about one month. This

[16] *Technical Memorandum No. 172–1*, U. S. Waterways Experiment Station, Vicksburg, Miss., January 4, 1941.

indicates that swelling of the bentonite seal did not create excess pore pressures in the clay. It seems entirely possible for pore-water pressures to become somewhat greater than total overburden pressures over small localized areas, especially in the case where the clay stratum is overlain by sand of appreciable thickness. The upward movement of the column of sand overlying the pressure concentration would be resisted not only by the weight of the column but also by friction along its sides.

Both Professor Terzaghi and Mr. Middlebrooks think that the computations of shearing resistance should have been based upon the results of fully consolidated shear tests rather than "consolidated-quick" tests. The writers are inclined to agree with this viewpoint. Although no such tests on the Pendleton clay are available, slow triaxal shear tests on other clays have shown that the "failure envelope" is a straight line passing through the origin (cohesive strength of zero) and having an inclination of about 30°. If cohesive strength of zero and a true angle of internal friction of 30° is assumed for the Pendleton clay, the computed factors of safety of the land-side portion of the foundation remain practically unchanged; but those for the river side become smaller. The writers hope to investigate this matter more thoroughly at the earliest opportunity.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

UNUSUAL CUTOFF PROBLEMS—DAMS OF THE TENNESSEE VALLEY AUTHORITY
A SYMPOSIUM

Discussion

By L. C. Glenn

L. C. Glenn,[11] Esq.[11a]—The emphasis in each of the papers in this Symposium is twofold: (1) Geological conditions, which, in general, were formerly considered as unusual, are described; and (2) the ingenious methods devised to cope with the problems are detailed. The extensive explorations, by the TVA, of dam sites in limestones have thoroughly discredited the old idea that circulation and consequent solution quickly cease below ground-water level. Solution under favorable circumstances may extend several hundred feet below ground-water level. Fortunately, solution does have a tendency to decrease in depth, so that solution channels may be expected to close up gradually if followed down with the necessary persistence. At Kentucky Dam the ordinary solutional activity below ground water has undoubtedly been considerably augmented by a depression of the water table in this region of some 200 ft or more in early Pleistocene time. The Ohio River gorge, only about 25 miles away, has been cut down to near sea level and has since filled with sand and gravel. In one of the very deep 48-in. holes at Kentucky Dam, pieces of chert were found near the bottom encrusted with limonite. The surfaces of the pieces of chert were covered with fine limonitic stalactites about an inch in length. These could have been formed only when the chert was above ground-water level. Since then, there has been a regional subsidence of some 200 to 300 ft at least. During the higher stand of the land there was a deeper-reaching solution.

Rocks at Kentucky Dam have been subjected to two periods of stress, each of which produced jointing or fracturing. The earlier fractures were later healed by deposition of coarsely crystalline calcite. The later fractures were

Note.—This Symposium was published in November, 1943, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: February, 1944, by Berlen C. Moneymaker; and April, 1944, by Portland P. Fox, and J. K. Black.

[11] Cons. Geologist, Nashville, Tenn. (formerly Head, Dept. of Geology, Vanderbilt Univ., Nashville, Tenn.).

[11a] Received by the Secretary March 29, 1944.

not healed subsequently but have become the paths for the solution that has given trouble. Incidentally, the same two periods of fracturing also characterize the rocks of the Ozarks, the earlier fractures being healed by calcite, dolomite, and occasionally sphalerite, as is shown by core drilling for dams in that region. The regional structural history has been similar.

Fig. 9 shows the marked tendency of grout to follow the contact between limestone and residual clay and how difficult it is to obtain any worth-while penetration of the grout into the clay. Mining the clay out, as was done at Kentucky Dam, is really the only effective method of treatment.

It is hoped that Mr. Schmidt has written finis to the long-standing leakage troubles at Hales Bar. Water flowing in strong currents through honeycombed limestone beneath the dam was constantly enlarging its passageways. Also, the water velocity added much to the difficulty of successfully placing concrete to stop the flow and to consolidate the foundation. One is sometimes inclined to wonder how the dam has stood up at all, especially in recent years. Mr. Schmidt describes well the intricate details of the plan adopted for closure and the modifications necessary to meet conditions that developed during the work. He has shown himself highly ingenious and resourceful in handling a very difficult problem and in solving it in an original and effective way. Although some leakage may develop in the future, the troubles at Hales Bar seem essentially to belong to the past.